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Porosity of the Two Ration Cantor Set

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Abstract

In this note we prove that the two ration Cantor set is uniformly very porous in the set of real numbers.

1. Introduction

Let r_1 and r_2 be positive real numbers such that $r_1+r_2 <1$. Put

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$$E_0 = [0,1],$$
$$E_1 = [0, r_1] \cup [1 - r_2, 1],$$
$$E_2 [0, r_1^2] \cup [r_1(1 - r_2), r_1] \cup [1 - r_{2,} 1 - r_2(1 - r_2)] \cup [1 - r_1 r_2, 1],$$

and so on, For any positive integer k, E_k is a union of 2k disjoint closed intervals, If [a,b] is a typical interval in Ek then the intervals in E_{k+1} are given by [r_1a,r_1b] and [r_1a,r_1b]. Further if c be such that r_1<c<1-r_2 then all intervals [r_1a,r_1b] lie to the left of c and all intervals [1-r_2b, r-r_2a] lie to the right of c. Clearly $E_{k+1} \subset E_k$. also note that if J is any interval in E_{k+1}, then there is a unique interval I in E_k such that $J \subseteq I$ and moreover

length (*J*) = r_1 length (*I*) or length(*J*) = $r_2$length(*I*) (1.1)

We set

$$C(r_1, r_2) = \bigcap_{k=0}^{\infty} E_k .$$

The set C(r_1,r_2) has been called by Coppel the two ratio Cantor set. If r_1 =r_2 =1 then C(r_1,r_2) coincides with the classical Cantor set, C. In this note we like to discuss the proposity behaviour of C(r_1,r_2).

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2. Some Basic facts about Porosity

We recall the following basic facts about porosity. Let $M \subset R$ and let y be a real number. Let $\nu(y,r,M) = \sup\{t > 0; there is a \ z \in (y-r, y+r)$ suchthat $(z-t, z+t) \subset (y-r, y+r)$ and $M \bigcap (z-t, z+t) = \phi\}$

Note that if M is dense then $\nu(y,r,M) = -\infty$.

Let

$$\overline{P}(y,M) = \limsup_{r \to 0+} \frac{\nu(y,r,M)}{r},$$

$$\underline{P}(y,M) = \liminf_{r \to 0+} \frac{\nu(y,r,M)}{r},$$

and if $\overline{p}(y,M) = \underline{p}(y,M)$ then we set

$$p(y,M) = \underline{p}(y,M) = \overline{p}(y,M) = \lim_{r \to 0+} \frac{\nu(y,r,M)}{r}$$

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The set M is said to be porous at y if p(y,M)>0 and $\sigma-porous$ at y provided $M=\bigcup_{n=1}^{\infty}$ M_n and each of the sets M_n is porous at y. M is called porous or $\sigma-porous$ set if it is so at each $y\in R.$

A porous set is nowhere dense while a $\sigma-porous$ set is a set of first category. Very simple examples of porous sets are the set of natural numbers and the set of all integers as also the classical Cantor set. The set of all retional numbers is an example of a $\sigma-porous$ set which is not porous. However not all nowhere dense set are porous and there are example of set which are nowhere dense but not porous.

The set M is said to be very porous at y if p(y,M) > 0 and M is said to be uniformly very porous in $Y_0\subset R$ if there is a c >0 such that for eaxh $y\in y_0$ we have $\underline{p}(y,M)\geq c.$

A very porous set or a uniformly very porous set is evidently porous but the converse is not generally true.

As already mentioned, for the classical Cantor set C, it is known that C is porous at each point of C. We prove here a stronger result for $C(r_1,r_2)$ in respect of porosity. Incidentally, on taking $r_1=r_2=\frac{1}{3}$ our theorem extends the theorem on porosity of C.

3. Theorem

Theorem 3.1. The set $C(r_1,r_2)$ is uniformly very porous in R.

Proof : if, $x\notin C(r_1,r_2)$ then clearly $\underline{p}[x,C(r_1r_2)]=1.$ So let $x\in C(r_1,r_2).$ then $x\in E_k$ for allnon-negative integers k. Let us take I_0 = [0,1] and t_0 = length(I_0) = 1. Since E_k is the union of disjoint closed intervals, there exists such an interval Ik from Ek that $x\in I_k$ and this is true for each k.

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Thus $\{I_k\}$ is a strictly decreasing sequence of closed intervals with length(Ik) = t_k (say) tending to zero as $k \to \infty$. Let r > 0. Without any loss of generality we choose r <1. Select a positive integer m such that. $t_{m+1} < r \le t_m$. it follows that $t_{m+1} = r_1 t_m$ or $t_{m+1} = r_2 t_m$.

Now since $x \in I_{m+1}$ and r>tm+l = length(Im+l), we must have $(x-r, x+r) \supset I_{m+1}$. The Interval I_{m+1} contains an interval of length $(1-r_l-r_2)t_{m+l}$ which does not contain any point of $C(r_1,r_2)$. Choose $t = \frac{t_{m+1}}{2}(1-r_l-r_2)$. Then $\frac{1}{2}\nu[x,r,C(r_1,r_2)] \ge \frac{1}{r} \ge \frac{1}{t_m} = \frac{(1-r_l-r_2)t_{m+1}}{2t_{m+1}} \ge \frac{(1-r_l-r_2)}{2}\min(r_1,r_2) = c$ say.

Thus c is positive and

$$\underline{p}[x, C(r_1,r_2)] = \underset{r\to 0+}{lin\inf} \frac{\nu[x,r,C(r_1,r_2)]}{r} \ge c.$$

This shows that $C(r_1,r_2)$ is uniformly very porous in R and thus the theorem is proved.

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