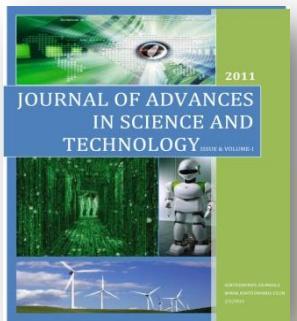


# Structural Health Conducting in Clever Structures by Lamb Waves Approaches



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**Abstract**

Nowadays there is great interest in structural damage detection using non-destructive tests. Once the failure is identified, as for instance a crack, it is possible to plan the next step based on a predictive maintenance program. There are several different approaches that can be used to obtain information about the existence, location and extension of the fault in mechanical systems by non destructive tests. This paper presents a technique for structural health conducting (SHM) based on Lamb waves approach using piezoelectric material as actuators and sensors. Lamb waves are a form of elastic perturbation that remains guided between two parallel free surfaces, such as the upper and lower surfaces of a plate, beam or shell. Lamb waves are formed when the actuator excites the surface of the structure with a pulse after receiving a signal. In this context, a flexible plate using three PZT actuators and three PZT sensors was used to make the configuration of the Lamb waves approach. The aluminum plate was represented by a model of second order written in modal coordinates. Structural damages were simulated through reduction of the stiffness in one

element. The results showed with clarity the location of the simulated damage; so, proving the viability of the presented methodology.

## Introduction:

Structural health conducting (SHM) is an emerging research area with multiple applications. SHM assesses the state of structural health and, through appropriate data processing and interpretation, predicts the remaining life of the structure. Many aerospace and civil infrastructure systems are at or beyond their design life; however, it is envisioned that they will remain in service for an extended period. SHM is one of the enabling technologies that will make this possible. Another potential SHM application is in new systems. By embedding SHM sensors and sensory systems into a new structure, the design paradigm can be changed and considerable savings in weight, size, and cost can be achieved. Not surprisingly, the SHM of aerospace structures has recently received increased attention [1 – 4].

There is a large number of non-destructive evaluation (NDE), non-destructive testing (NDT), and non-destructive inspection (NDI) techniques for identifying local damage and detect incipient failure in critical structures. Among them, ultrasonic inspection is well established and has been used in the engineering community for several decades [5]. In an infinite solid medium, elastic waves can propagate in two basic modes: pressure (P) waves and shear (S) waves. However, if the medium is bounded, wave reflections occur at the boundary and more complicated wave patterns emerge. Of particular interest are the guided waves, which remain contained in a wave guide and can travel at large distances. Lamb waves are guided waves traveling along thin plates and can be generated by transducers. Guided waves are being used to detect cracks, inclusions, and disbonds in metallic and composite structures. Lamb waves are appropriate for thin plate and shell structures.

SHM sets out to determine the health of a structure by reading a network of sensors that are embedded (permanently attached) into the structure and conducted over time. SHM can be either passive or active. Passive SHM infers the state of the structure using passive sensors that are conducted over time and fed into a structural model. Active SHM uses active sensors that interrogate the structure to detect the presence of damage, and to estimate its extent and intensity. One active SHM method employs piezoelectric wafer active sensors (PWAS), which send and receive Lamb waves and determine the presence of cracks, delaminations, disbonds, and corrosion.

Many researchers have studied the technology of SHM integrating piezoelectric sensors/actuators into the structures. Using these actuators and sensors is possible to realize a structural conducting through Lamb waves approach. Lamb waves have been used in ultrasonic testing and material evaluation for several decades, and numerous research endeavors have been undertaken to study feasible methods of generating and receiving these kinds of waves. For the *in situ* or *in service* health conducting of critical structures used in aerospace engineering, Lamb waves cannot be directly applied in conventional testing techniques [6], because they usually require bulky instruments and human interference [7].

Several different applications can be cited in this area. Since they can explore the entire thickness of a plate in a single interrogation, Lamb waves are particularly well adapted to the aerospace industry, ground transportation, or even civil engineering, where the use of composites is becoming increasingly common. For instance, Chimenti and Martin [8] originally used Lamb waves to detect various defects, such as delamination, porosity, ply gap, presence of foreign material, and changes in fiber volume fraction of carbon/epoxy laminates. Tan et al. [9] compared Lamb waves with the normal incidence pulse-echo approach for the detection of near-surface delaminations. Changes in the Lamb wave amplitude are used to determine both the size and the depth of the delamination. Kessler et al. [10] used piezoelectric actuators to provide Lamb wave scans in damage detection in various composite structures (laminates, sandwich beams, pipes, and

stiffened plates). It showed that for localized flaws, Lamb waves enable the retrieval of richer information about damage type, location, and extent than frequency response techniques. The high sensitivity of pulse-echo signals to the presence of cracks in aluminum plates was demonstrated by the simple subtraction of the baseline obtained in the pristine state. This type of result, obtained by measuring amplitude of Lamb waves signature with piezoelectric transducers [11], is favored by the use of simple material and the high signal-to-noise ratio of in-the-laboratory experiment. Outdoor tests on real structures cannot satisfy these requirements and thus demand the development of more complex data analysis procedures (multivariate data processing, Fourier, and wavelet Transform, etc.). More recently, Diamanti et al. [12] attached a linear array of piezoceramic patches onto the surface of a composite structure to investigate the interaction of Lamb waves with impact damage in CFRP laminates over large areas.

Sohn et al. [13] proposed a multi-scale structural health conducting approach for detecting defects in composite structures by combining Lamb wave propagation, impedance-based methods and time reversal acoustics using a common active sensing system for local nondestructive evaluation. These authors mention that the ultrasonic research community has studied Lamb waves for the nondestructive evaluation of plates since the 1960s. They also define Lamb waves as mechanical waves corresponding to vibration modes of plates with a thickness of the same order of magnitude as the wavelength. In the study presented, the wave attenuation feature, identified using a wavelet based damage index, is used to locate the region of damage [13]. Damage was introduced by firing a small projectile at different locations on a graphite fibre/epoxy matrix square plate with varying velocities. It was found that the three methods studied can be complementary because they can use the same sensors/actuators, which in this case were commercially available thin films with embedded PZT sensors. For instance, while it was observed that the Lamb wave propagation method is effective for thin plates, the impedance method is more suitable for detecting damage near structural joints or connections.

As a result of the varying challenges offered by different structures and systems, significant research effort has been applied to condition conducting with the emergence of a broad range of techniques, algorithms and methods. Rytter [14] classified the various methods based on the level of identification attempted:

Level 1: Determination that damage is present in the structure.

Level 2: Determination of the geometric location of the damage

Level 3: Quantification of the severity of the damage

Level 4: Prediction of the remaining service life of the structure

In this context, this paper presents a technique of structural health conducting using the level 2 of the damage identification. The tests are realized in a flexible plate using three PZT actuators and three PZT sensors to make the configuration of the Lamb waves approach. The aluminum plate was represented by a model of second order written in modal coordinates. Structural damages were simulated through reduction of the stiffness in one element. The results showed with clarity the location of the simulated damage; so, proving the viability of the presented methodology

## **Structural Modeling:**

The approaches are demonstrated theoretically through an analytical model. It is possible to describe the dynamical behaviour of a structure in terms of mass, stiffness and damping matrices, and displacement and velocity vectors as

$$\ddot{\mathbf{q}}(t) + \mathbf{M}^{-1} \mathbf{D}_a \dot{\mathbf{q}}(t) + \mathbf{M}^{-1} \mathbf{K} \mathbf{q}(t) = \mathbf{M}^{-1} \mathbf{B}_0 \mathbf{u}(t) \quad (1)$$

$$\mathbf{y}(t) = \mathbf{C}_{oq} \mathbf{q}(t) + \mathbf{C}_{ov} \dot{\mathbf{q}}(t)$$

where  $\mathbf{q}(t)$  is the  $n$ -length displacement vector,  $\mathbf{u}(t)$  is the  $s$ -length input vector,  $\mathbf{y}(t)$  is  $r$ -length output vector,  $\mathbf{M}$  is the  $n \times n$  mass matrix,  $\mathbf{D}_a$  is the  $n \times n$  damping matrix, and  $\mathbf{K}$  is the  $n \times n$  stiffness matrix.  $\mathbf{B}_0$  is the  $n \times s$  input matrix,  $\mathbf{C}_{oq}$  is the  $r \times n$  output

displacement matrix, and  $\mathbf{C}_{ov}$  is the  $r \times n$  output velocity matrix. The mass matrix is positive definite, and the stiffness and damping matrices are positive semi-definite,  $n$  is the number of degrees of freedom of the system (linearly independent coordinates describing the finite-dimensional structure),  $r$  is the number of outputs and  $s$  is the number of inputs. Using the classic procedure of modal analysis [15], it is possible to write the equations of motion in modal coordinates,  $\mathbf{q}_m(t)$ . Thus, the modal model of second order is given by



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$$\mathbf{q}(t) = \Phi \mathbf{q}_m(t) \quad (2)$$

$$\ddot{\mathbf{q}}_m(t) + 2\mathbf{Z}\Omega\dot{\mathbf{q}}_m(t) + \Omega\mathbf{q}_m(t) = \mathbf{B}_m \mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}_{mq} \mathbf{q}_m(t) + \mathbf{C}_{mv} \dot{\mathbf{q}}_m(t)$$

where  $\Phi$  is the modal matrix and  $\mathbf{Z}$  is the matrix of damping coefficients ( $\zeta$ ), given by

$$\mathbf{Z} = 0.5 \mathbf{M}_m^{-1} \mathbf{D}_m \Omega^{-1} = 0.5 \mathbf{M}_m^{-1/2} \mathbf{K}_m^{-1/2} \mathbf{D}_m \quad (3)$$

where  $\Omega^2 = \mathbf{M}_m^{-1} \mathbf{K}_m$  is the matrix of natural frequencies. The matrices  $\mathbf{M}_m$ ,  $\mathbf{K}_m$  and  $\mathbf{D}_m$  are diagonal matrices of modal mass, stiffness and damping, respectively, which are given by

$$\begin{aligned}\mathbf{M}_m &= \Phi^T \mathbf{M} \Phi \\ \mathbf{K}_m &= \Phi^T \mathbf{K} \Phi \\ \mathbf{D}_m &= \Phi^T \mathbf{D}_a \Phi\end{aligned}\tag{4}$$

The matrix  $\mathbf{D}_a$  is assumed to be proportional to mass and stiffness matrices, so that

$$\mathbf{D}_a = \alpha \mathbf{M} + \beta \mathbf{K}\tag{5}$$

with  $\alpha$  and  $\beta$  constants. Matrix  $\mathbf{B}_m$  in equation (2b) is the input modal matrix, or participation modal matrix and is given by

$$\mathbf{B}_m = \mathbf{M}_m^{-1} \Phi^T \mathbf{B}_0\tag{6}$$

$\mathbf{C}_{mq}$  and  $\mathbf{C}_{mv}$  are the output displacement and velocity modal matrices given by

$$\begin{aligned}\mathbf{C}_{mq} &= \mathbf{C}_{eq} \Phi \\ \mathbf{C}_{mv} &= \mathbf{C}_{ov} \Phi\end{aligned}\tag{7}$$

The motion equations, eq. (2), can be written in state space form by vector-matrix format through the triple (**A**, **B**, **C**). It allows the equations to be manipulated more easily.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}\tag{8}$$

where **x**(t) is the state vector, **A** is the dynamic matrix, **B** is the input matrix, **u**(t) is the input force, **y**(t) is the output vector and **C** is the output matrix. The related matrices are given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}^2 & -2\mathbf{Z}\mathbf{\Omega} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_m \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{mq} & \mathbf{C}_{mv} \end{bmatrix} \tag{9}$$

Equation (8) is not a modal state representation (although it was obtained using modal displacements, **q**<sub>m</sub>). The modal state-space representation has a triple (**A**<sub>m</sub>, **B**<sub>m</sub>, **C**<sub>m</sub>) characterized by the block-diagonal dynamic matrix, **A**<sub>m</sub>, and the related input and output matrices [16]

$$\begin{aligned}\mathbf{A}_m &= \text{diag}(\mathbf{A}_{mi}) & \mathbf{B}_m &= \begin{bmatrix} \mathbf{B}_{m1} \\ \mathbf{B}_{m2} \\ \vdots \\ \mathbf{B}_{mn} \end{bmatrix}, \\ \mathbf{C}_m &= \begin{bmatrix} \mathbf{C}_{m1} & \mathbf{C}_{m2} & \cdots & \mathbf{C}_{mn} \end{bmatrix}\end{aligned}\tag{10}$$

where  $i=1,2,\dots,n$ ;  $\mathbf{A}_{mi}$ ,  $\mathbf{B}_{mi}$  and  $\mathbf{C}_{mi}$  are  $2 \times 2$ ,  $2 \times s$  and  $r \times 2$  blocks, respectively. These blocks can take several different forms and also it is possible to convert from one form to another by a linear transformation. One possible form to block  $\mathbf{A}_{mi}$  is:

$$\mathbf{A}_{mi} = \begin{bmatrix} -\zeta_i \omega_i & \omega_i \\ -\omega_i (\zeta_i^2 - 1) & -\zeta_i \omega_i \end{bmatrix} \quad (11)$$

The state vector  $\mathbf{x}(t)$  in modal coordinates consists of  $n$  independent components,  $\mathbf{x}_i(t)$ , that represent a state of each mode. The  $\mathbf{x}_i(t)$  (ith state component), related to equation (11), is given by [4].

$$\mathbf{x}_i(t) = \begin{Bmatrix} \mathbf{q}_{mi}(t) \\ \mathbf{q}_{moi}(t) \end{Bmatrix}, \quad \mathbf{q}_{moi}(t) = \zeta_i \mathbf{q}_{mi}(t) + \dot{\mathbf{q}}_{mi}(t) / \omega_i \quad (12)$$

### Damage Detection Based On Lamb Waves:

Structural health conducting system based on Lamb waves technique includes a sensors network for data acquisition and some internal processors employing an algorithmic for evaluating the structural conditions. The use of piezoelectric actuators coupled at the surface of a plate forming the Lamb waves is an increasing area of studies. Lamb waves are formed when the actuator excites the surface with a pulse after receiving a signal. Another set of piezoelectric ceramic is used as sensor for receiving the pulses, which comes from actuator. When PZTs are used as actuators and coupled in the plate surface it tends contracting or expanding, depending of the polarity of the electrical field applied. The structure strains and a fold movement are

introduced in the surface and the waves propagate in the plate. These waves are “felt” for another PZT’s, now used as sensors [17].

When a wave propagates in the plate, it arrives in a sensor PZT browsing different ways. One of the ways is when the wave arrives directly at the sensor, without obstacles in the way where the wave propagated. The other possible way is when the wave arrives at the sensor after reflected in contours or discontinuities contained in the surface (structural imperfections). These ways are known and the damages can be located in these ways. With the several characteristics of the received signal, such as delay in the time of passage, frequency, amplitude, and with the use of certain techniques of signals processing, such as artificial neural networks, information about the damage can be obtained.

Figure 1 shows two sets of PZT’s placed in the plate surface. A set of PZT’s is used as actuator, sending a predefined wave through the plate surface. Thus, the other set of PZT’s (adjacent) becomes sensor and measurer of the reply signals. The PZT’s setup in the plate is shown in Fig. 1; and the procedure of excitation and measuring of the signals are shown in the Fig. 2. The scheme of actuators-sensors formed in the plate is known as Piezoelectric Wafer Active Sensors (PWAS) and has been very important for the implementation and the development of SHM methodologies [18].

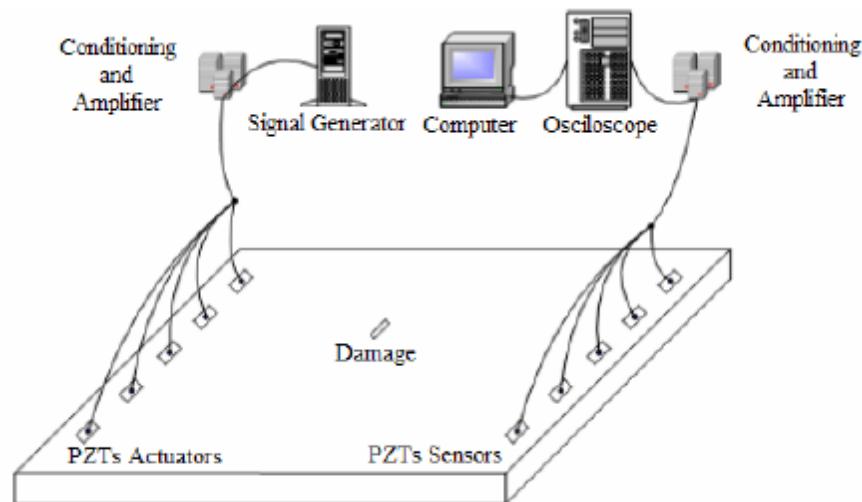


Figure 1. Setup of a SHM system using Lamb waves approach.

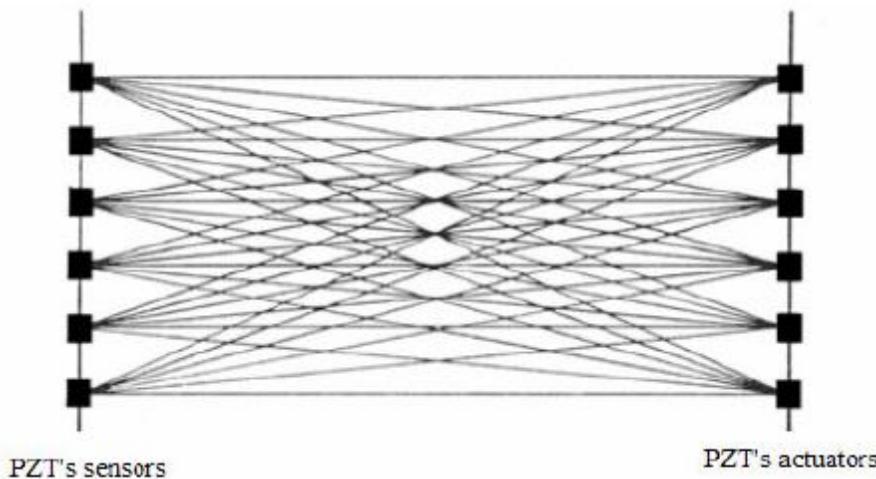


Figure 2. A layout of PZT's sensors/actuators.

The location of damages, in this methodology, is based on a comparison between the frequency response function (FRF) of the system before and after the damage. The FRF can be

compared using the metric damage index that supplies an estimate of the structural variation, as shown in the following equation

$$M = \sum_{i=1}^n [(Y_{i,1}) - (Y_{i,2})]^2 \quad (13)$$

where  $M$  is the metric damage index,  $Y_{i,1}$  is the FRF magnitude measured in the structure before the damage,  $Y_{i,2}$  is FRF magnitude measured in the structure in normal operation conditions (after the damage); and  $n$  is the number of points used in the acquisition of the signal [19].

## Numerical Application:

The proposed methodology was numerically applied in an aluminum plate structure, as shown in Fig. 3. The physics and geometric properties of the plate are given in Table 1. The boundary conditions were clamped-free-free-free. The plate is discretized by Finite Element Method (FEM) [20] in 100 elements and 363 structural dof's (121 nodes). The plate is clamped in one end, so considering this boundary condition, the system has  $N = 660$  states.

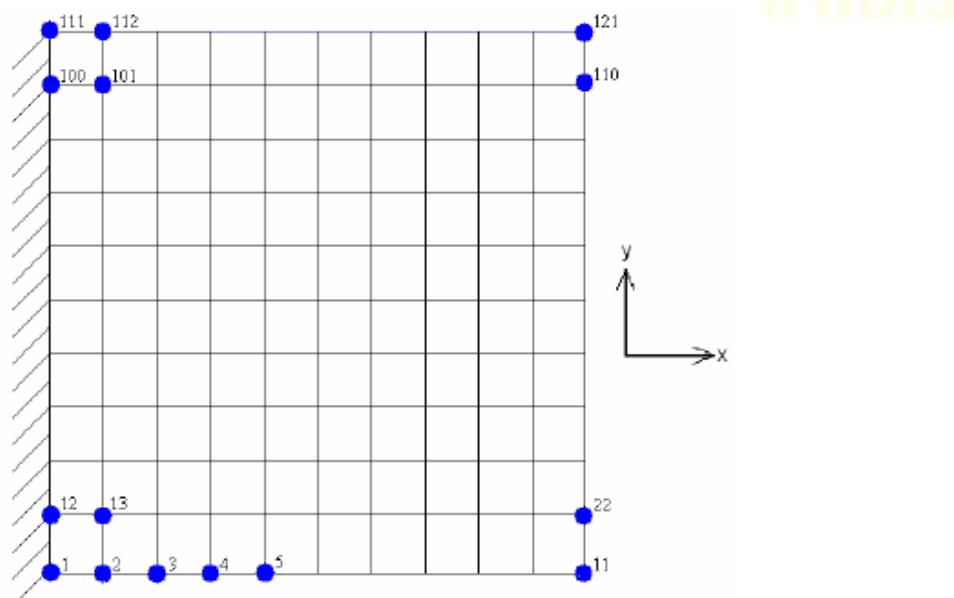


Figure 3. Finite element model for a cantilever plate.

Property	Value
Young's Modulus (GPa)	70
Poisson's Coefficient	0.3
Density (kg.m <sup>-3</sup> )	2710
Length (mm)	200
Width (mm)	5
Thickness (mm)	3

Table 1. Geometric and physic properties of the plate.

One damage case was analyzed considering 10% of reduction in the stiffness of the element 22 (Fig. 4). The concept of this methodology requires measurement points in the region under analysis. For practical situations is not necessary the conducting of every element. The engineer must choose the probable positions to occur damages. In the following application, with the objective of verifying the sensitivity of each sensor position, was considered PZT sensors placed in elements 2, 6 and 9 and PZT actuators placed in elements 72, 76 and 79, as shown in Fig. 4.

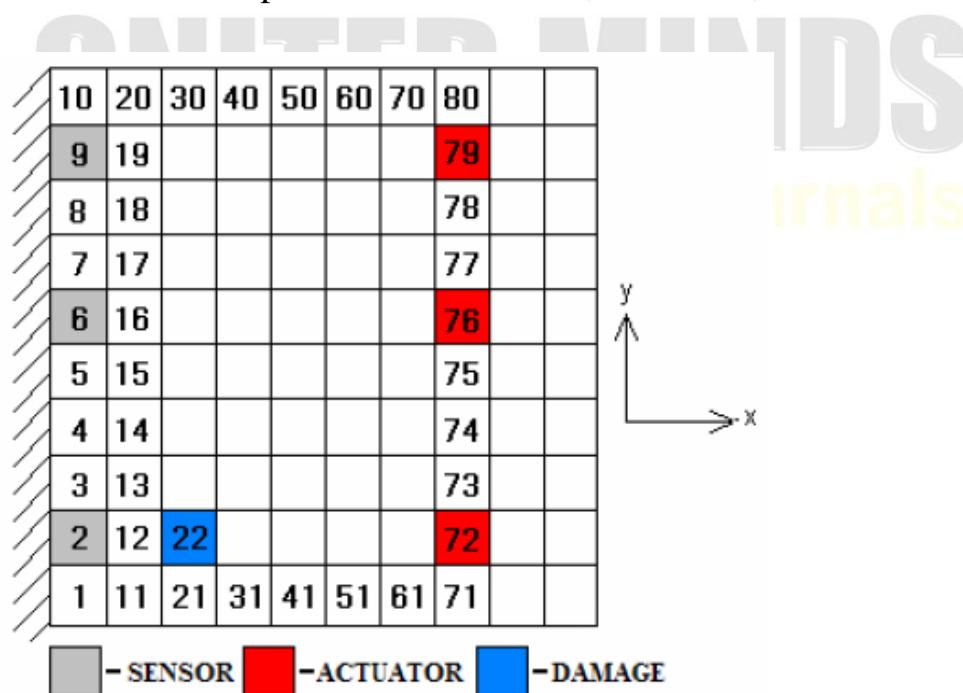


Figure 4. Schematic picture showing the position of three PZTs sensors, three PZTs actuators and of the damage.

Initially, the structure was excited with PZT actuator 1 and the signals were measured by sensors 1, 2 and 3, as shown in Fig. 5. The PZT actuators were not excited simultaneously. Figure 6 shows the Frequency Response Function (FRF) for each input/output, considering the first four vibration modes.

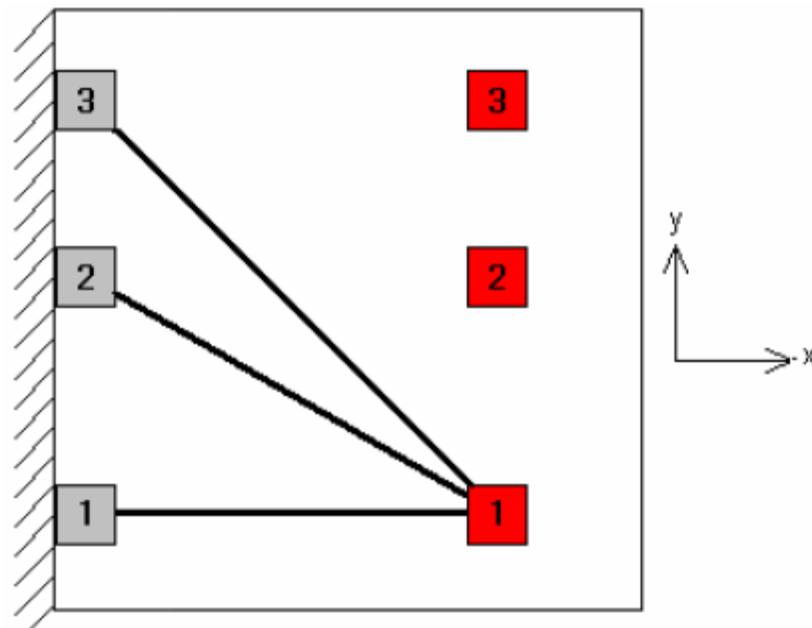


Figure 5. First PZT actuator exciting the structure.

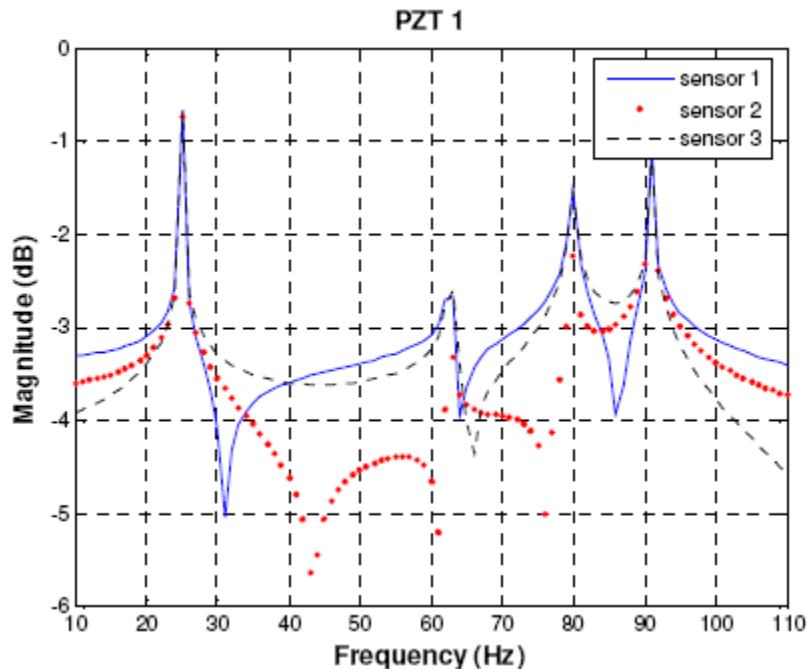


Figure 6. FRF of the system considering each PZT sensor and the PZT actuator 1 – before the damage.

Figures 7 and 9 show the schematic picture of the plate excited with PZT actuators 2 and 3, respectively. While, figures 8 and 10 show the respective FRFs.

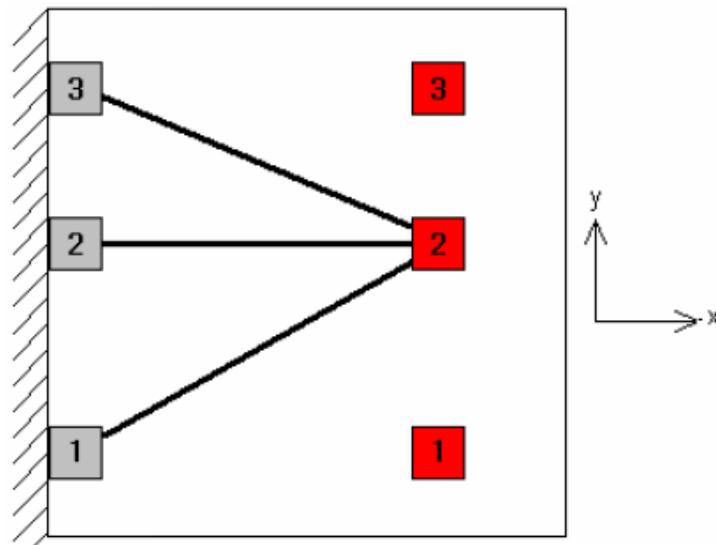


Figure 7. PZT actuator 2 exciting the structure.

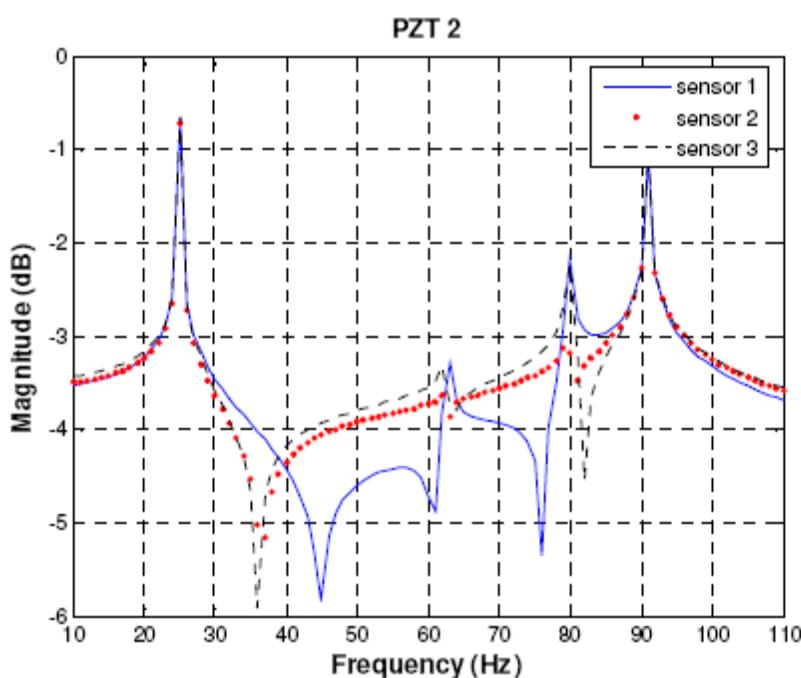


Figure 8. FRF of the system considering each PZT sensor and the PZT actuator 2 – before the damage.

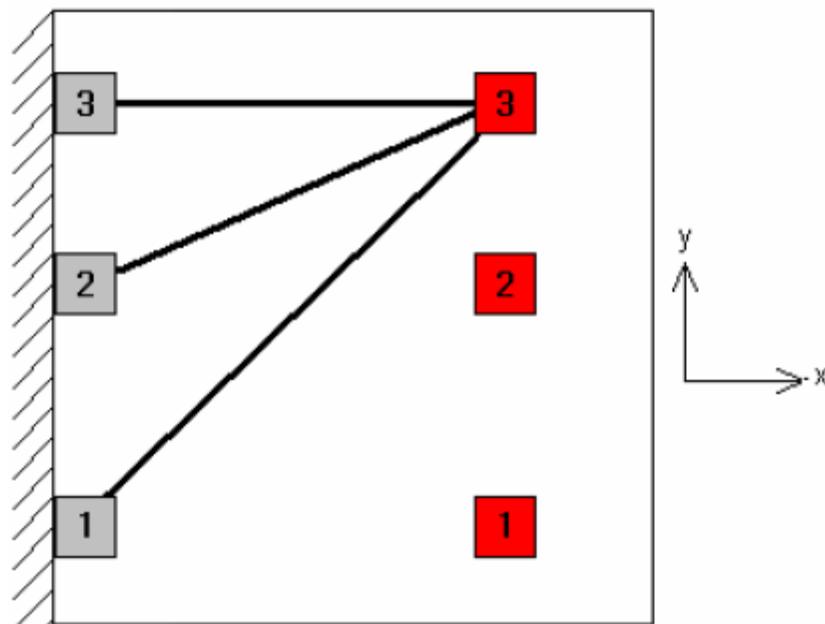


Figure 9. PZT actuator 3 exciting the structure.

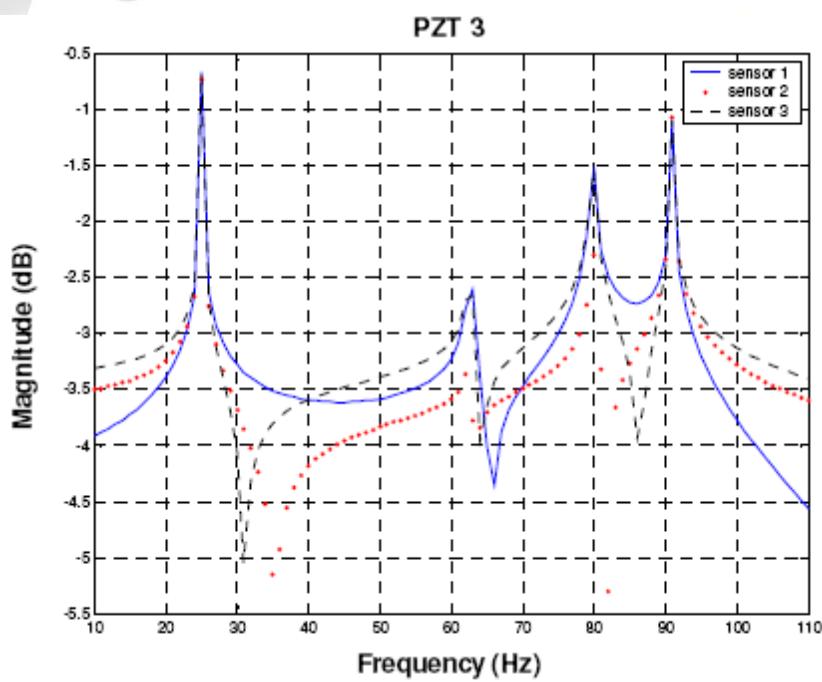


Figure 10. FRF of the system considering each PZT sensor and the PZT actuator 3 – before the damage.

The same procedure was repeated after the introduction of the damage, i.e., a reduction of 10% in the stiffness of the element 6. Figures 11, 12 and 13 show the FRF of the system with damage for each PZT actuator, respectively.

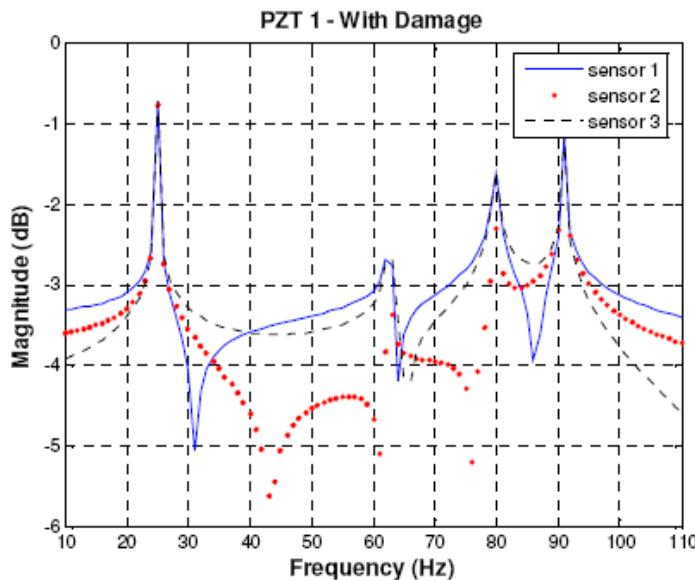


Figure 11. FRF of the system considering each PZT sensor and the PZT actuator 1 – after the damage.

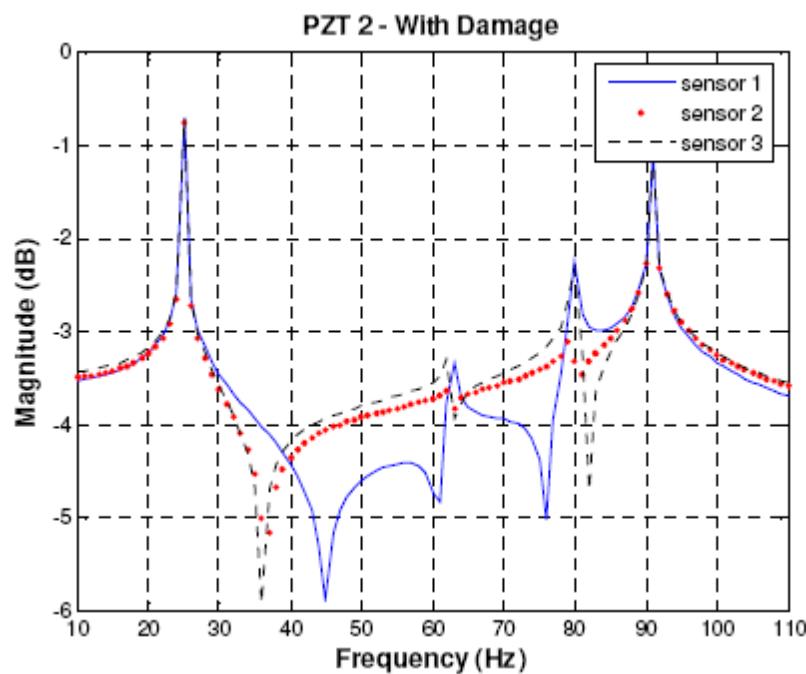


Figure 12. FRF of the system considering each PZT sensor and the PZT actuator 2 – after the damage.

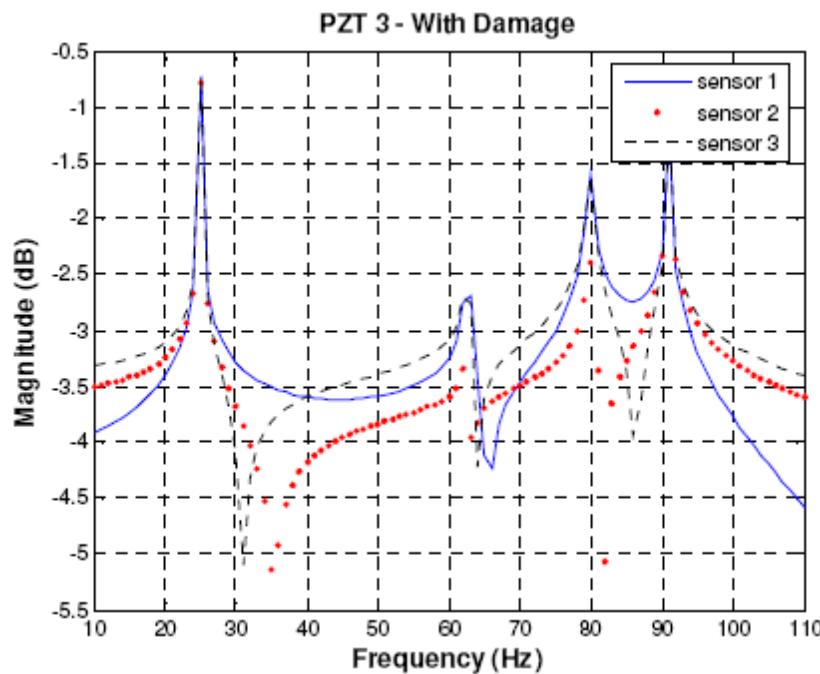


Figure 13. FRF of the system considering each PZT sensor and the PZT actuator 3 – after the damage.

For the data acquisition 100 points had been used, (from 10 to 110 Hz). Figure 14 shows the normalized metric damage index obtained considering this damage situation. Table 2 shows the values for a best comprehension of the results.

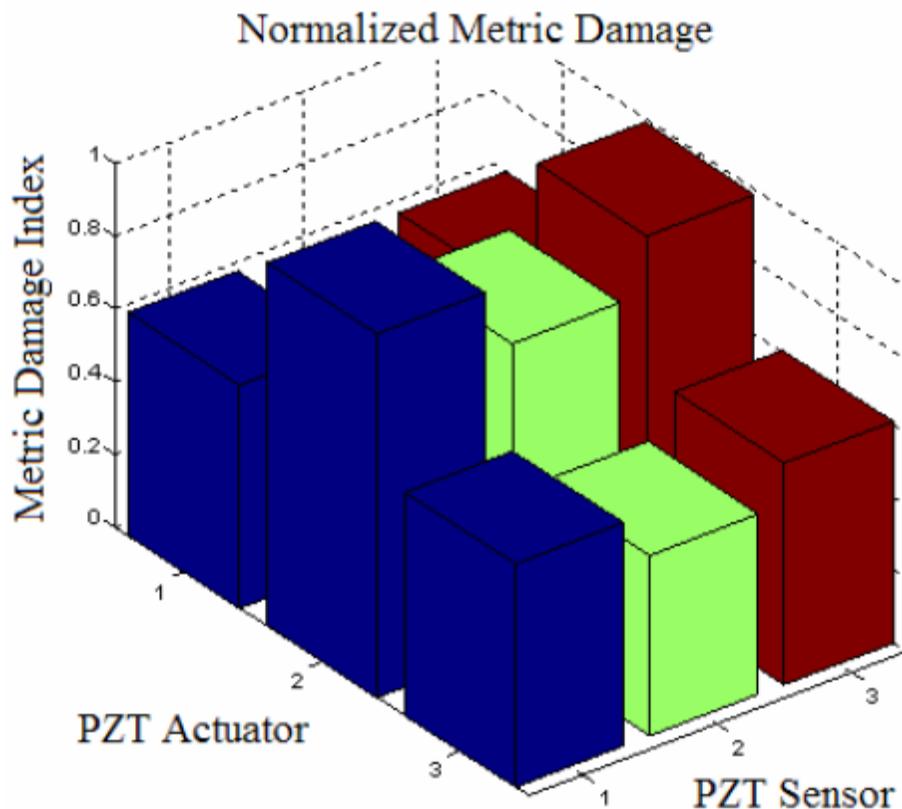


Figure 14. Metric damage indices considering the damage in element 6.

	Sensor 1	Sensor 2	Sensor 3
<b>Actuator 1</b>	0,0240	0,0194	0,0237
<b>Actuator 2</b>	0,0392	0,0326	0,0386
<b>Actuator 3</b>	0,0240	0,0195	0,0237

Table 2. Metric damage indices.

It is possible to observe that the more affected ways were: actuator 1 to sensor 1, actuator 2 to sensor 1 and actuator 3 to sensor 1. Making the intersection of these ways, it is possible to find the region of the damage identified. Figure (15) shows this region.

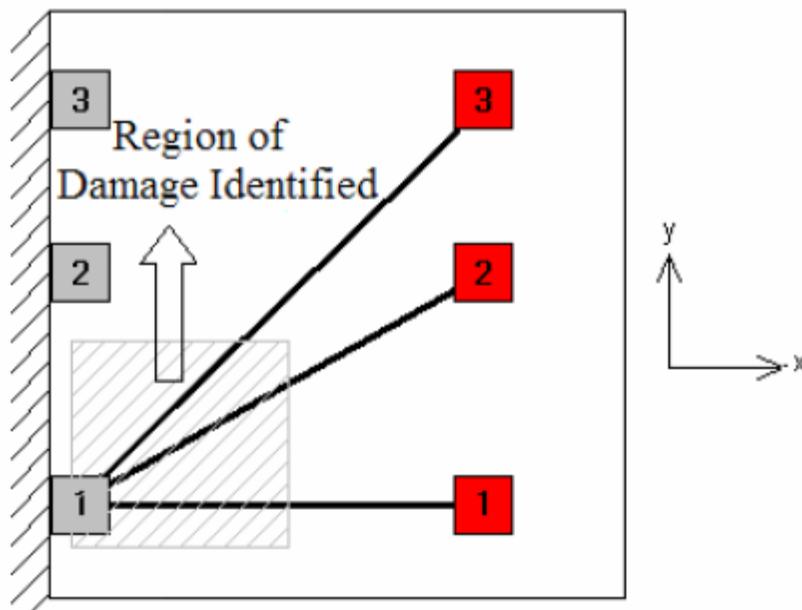


Figure 15. Localization of the damage region considering the indices with bigger variation.

### Final Remarks:

A time reversal concept was extended to Lamb wave propagations for detecting defects in a plate structure. The enhanced time reversal method is employed to improve the time reversibility of Lamb waves. A carefully designed narrowband waveform must be used to address the frequency dependency of the time reversal operator and an automated signal selection process can retain only a segment of the raw response signal that is more sensitive to damage and less responsive to changing boundary conditions.

In this work was shown a method of damage location using Lamb wave method. This methodology uses sensors and actuators coupled in the plate forming the PWAS. Incipient damages are a very important issue, and it can be conducted through the metric damage index for high frequencies. The presented work used FEM to build the numerical model, so, there was a limitation in the frequency range. Further works will be addressed for analysis of the distance

between the PZTs actuators and sensors and the FRF acquisition, considering different frequency sample rate. Also, the analytical model of the system can be obtained using spectral element method that is more adequate to evaluate the high frequency vibration modes.

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