**REVIEW OF SOFT COMPUTING METHODS IN GEOPHYSICAL APPLICATIONS**

**A.Stanley** $Raj^{1}$**, Y. Srinivas2, R.**$Damodharan^{2}$**, M.Sanjay Vimal2**

$ ^{1}$Department of Physics, Loyola College, Chennai-34.

$ ^{2}$Center for Geotechnology, Manonmaniam Sundaranar University, Tirunelveli-12.

Email: stanleyraj@loyolacollege.edu, drysv@yahoo.co.in, damodharanphy@gmail.com, sanjayvimal49@gmail.com

**ABSTRACT**

 Geophysics as its name indicates it has to do with Physics of the Earth. Geophysical prospecting methods involves exploration of interior of the Earth or the subsurface of the Earth. There are various Geophysical prospecting methods like Gravity prospecting, Magnetic prospecting, seismic prospecting, Electrical and Electromagnetic prospecting methods. Data obtained through these Geophysical prospecting methods are only indirectly related to the physical properties of the Earth, so an inverse problem must be solved to obtain estimates of the physical properties within the Earth. As the field data acquired is more non-linear it is necessary to invert the data using smart tools. Soft computing methods are more prominent in handling such data and researchers found it more successful in their applications (Srinivas et al., 2012, Stanley Raj et al., 2014, 2016). Soft computing based geophysical inversion differs from conventional computing in fixing uncertainty problems. Several Soft computing methods viz., Neural Networks, Fuzzy Logic, Neuro Fuzzy, Wavelet analysis, Cellular Automata, Nature inspired artificial optimization algorithms are applied to inverting most of the Geophysical methods. A complete overview of the soft computing methods applied in the field of Geophysics is presented in this paper. Research works based on these methods are studied, analyzed and discussed.

**Keywords:**

Neural Networks, Fuzzy, Neuro Fuzzy, Soft computing methods, Geoelctrical inversion

**1.Introduction:**

**1.1 Electrical methods**

 Geoelectrical resistivity method is used to investigate groundwater resources and its vulnerability of aquifers. The vulnerability depends on the geological structures of certain regions. The clayey structural formations show the low resitivities and sandy permeable formations showing high resistivities. Thus geoelectrical method is a powerful tool to investigate low and high resistive formations and therefore it is a powerful tool for studying vulnerable conditions in the study area (Chistensen & Sørensen 1998, Sørensen et al. 2005).

**Theory**

 Data obtained from resistivity surveys are apparent resisitivity data ρ*a*. Apparent resistivity is defined as the data obtained form homogeneous and isotropic half-space that represent the relationship between the applied current and the potential difference for aspecific arrangement of current electrodes. Equation governing the relation between current applied, distribution of electrodes. On considering the current electrode located on the boundary of semi-infinite, electrically homogeneous medium, which represents a homogeneous earth.  If the electrode carries a current I, measured in amperes (a), the potential at any point in the medium or on the boundary is given by:

 $U= ρ\frac{I}{2πr}$                                               (1)

where

*U* = potential, in *V,*

            ρ = resistivity of the medium,

            *r* = distance from the electrode.

For an electrode pair with current I at electrode A, and -I at electrode B (figure 1), the potential at a point is given by the algebraic sum of the individual contributions:

 $U= \frac{ρI}{2πr\_{A}}- \frac{ρI}{2πr\_{B}}= \frac{ρI}{2π} \left[\frac{1}{r\_{A}}-\frac{1}{r\_{B}}\right]$                                              (2)

where

*rA* and *rB* = distances from the point to electrodes *A* and *B*

Figure 1 illustrates the electric field around the two electrodes in terms of equipotentials and current lines.  The equipotentials represent imagery shells, or bowls, surrounding the current electrodes, and on any one of which the electrical potential is everywhere equal.  The current lines represent a sampling of the infinitely many paths followed by the current, paths that are defined by the condition that they must be everywhere normal to the equipotential surfaces.



Figure 1.  Equipotentials and current lines for a pair of current electrodes A and B on a homogeneous half-space.

In addition to current electrodes A and B, figure 1 shows a pair of electrodes M and N, which carry no current, but between which the potential difference V may be measured.  Following the previous equation, the potential difference *V* may be written

 $V= U\_{M}-U\_{N}= \frac{ρI}{2π} \left[\frac{1}{AM}-\frac{1}{BM}+\frac{1}{BN}-\frac{1}{AN}\right]$                            (3)

where

*UM* and *UN* = potentials at *M* and *N,*

                       *AM* = distance between electrodes *A* and *M*, etc.

These distances are always the actual distances between the respective electrodes, whether or not they lie on a line.  The quantity inside the brackets is a function only of the various electrode spacings.  The quantity is denoted 1/*K*, which allows rewriting the equation as:

 $V= \frac{ρI}{2π}\frac{1}{K}$                                                         (4)

where

*K* = array geometric factor.

 $ρ=2πK\frac{V}{I}$                                                            (5)

The resistivity of the medium can be found from measured values of *V*, *I*, and *K*, the geometric factor.  *K* is a function only of the geometry of the electrode arrangement.

**Apparent Resistivity**

Wherever these measurements are made over a real heterogeneous earth, as distinguished from the fictitious homogeneous half-space, the symbol ρ is replaced by ρ*a* for apparent resistivity.  The resistivity surveying problem is, reduced to its essence, the use of apparent resistivity values from field observations at various locations and with various electrode configurations to estimate the true resistivities of the several earth materials present at a site and to locate their boundaries spatially below the surface of the site.

Electrical conductivity of rocks is not the only attribute which is of value to exploration geologists. A number of different electrical properties of rocks are measured and interpreted in mineral exploration. They depend on:

a)      Natural currents in rocks – Self-potential method

b)      Polarizability of rocks – Induced polarization method

c)      Electrical conductivity or resistivity of rocks – Resistivity method

d)      Induction – Electromagnetic method

**Gravity Methods:**

 Gravity method is used to explore subsurface geology and it is relatively cheap, non-invasive, non-destructive technique. It requires no energy to acquire data and it can measure densities of rocks underground. Small portable carrying instrument well suited for taking reading in densely populated regions. The distribution of densities of rocks can be identified based on the mass underneath. It works on the basic concept of gravitational attraction exerted by the earth and the measurement station. Gravitational anomalies results from lateral distribution of subsurface materials especially the density of rock materials present underneath. The collected data is interpreted by finding the difference between observed gravity (gobs) and theoretical gravity (gthe) at any point on earth’s surface after deducting the corrections. Gravimeters have an accuracy of about 0.01 milligal (mgal; 1 mgal = 0.001 centimetre per second per second). Thus practically it can able to find the differences in the Earth's gravitational field as small as one part in 100,000,000.

**Magnetic methods:**

 Magnetic method is one of the modern exploration method useful in studying information related to the distribution of rocks, formation of subsurface layers and to identify mineral ores. Earth having its own magnetic field induces the rocks to possess magnetism. The strength of the magnetic field depends on the magnetic composition of the rock materials. The main magnetic mineral is magnetite (Fe3O4) - a common mineral found disseminated through most rocks in differing concentrations. Magnetometers are used to measure magnetic field of earth, other instruments are magnetic balances, fluxgate magnetometers, proton-precession and optical-pumping magnetometers. Sedimentary formations possess very low susceptibility value. Data obtained from igneous and metamorphic rock formations are useful to study the magnetic anomalies indicate the minerals concentrations. Contour map of the interpretations clearly reveals the ore concentration of the rocks.

**Seismic Methods:**

Seismic method is used to explore the subsurface parameters by calculating the time interval between the seismic wave generator and receiver. Artificial seismic vibrator can be used to generate seismic waves. Different methods of producing seismic waves artificially are by an explosion, a dropped weight, a mechanical vibrator, a bubble of high-pressure air injected into water, or other sources. It can be detected using Geophones to be located on the ground. Marine seismic surveys uses hydrophones to receive the signals generated from the source. Data recorded in the tape can be interpreted using softwares. There are two kinds of seismic methods used for surveys viz.,- Refraction methods and Reflection methods

Artificial Neural Networks:

Artificial Neural Networks plays vital role in extracting knowledge based informations. Computations involved in ANN has hidden nodes with connections and appropriate weights which forms the dense mesh to perform calculations (Jackek Zuradha, 2006). It is well proved noise immune and fault tolerant system which can capable of extracting information from previously learned examples (Sreekanth *et al.,* 2009). Thus it has been applied for modeling many geoelectrical inversions (Ahmad Neyamadpour *et al.,* 2010; Brown & Poulton, 1996; Constable *et al.,*1987; Dey & Morrison, 1979; Gad-el-Qady & Keisuke Ushijima, 2001; Griffith & Barker. 1993; Jimmy Stephen & Manoj Singh, 2004; Loke & Barker, 1996; Smith & Vozoff, 1984; Tripp *et al.,* 1984; Uchida & Murakami, 1990). Aquifer parameters such as thickness and resistivity can be studied by direct current resistivity methods mainly used in the field of geophysical exploration (Kosinky & Kelly, 1981; Mazac *et al.,* 1985; Yadav & Abolfazli, 1978). Vertical Electrical Sounding (VES) data can be interpreted using curve matching technique is well useful for estimating the subsurface geology (Flathe, 1955; Ghosh, 1971; Mooney *et al.,* 1966. Due to advancement in computational science the interpretation can be done by soft computing tools. Stanley Raj et al. 2014 applied Neural networks model for inverting geoelectrical data.

**4 ANN architecture**

Feed forward back propagation (FFBP) algorithm is one of the successful technique in training the given examples and testing the unknown parameters using the trained weights. The concept of FFBP applied here to reduce the electrical resistivity noises present in the field. Backward error reduction process will be successfully deone by adjusting and updating the weights in each iteration during the process of training.

The backpropagation algorithm updates neuronal activations in the network for the input layer as

δ($x\_{i}^{k}$) =$ x\_{i}^{k}$, i=1,……., n (1)

δ($x\_{0}^{k}$) = $x\_{0}^{k}$ = 1 (2)

Where $x\_{i}^{k}$ is the ith component of the input vector presented in the network, and δ($x\_{0}^{k}$) is the input layer bias neuron signal that is independent of iteration index.

And for the hidden layer the network activation will be

 $z\_{h}^{k}$= $\sum\_{i=0}^{n}w\_{ih}^{k}$ δ ($x\_{i}^{k}$) = $\sum\_{i=0}^{n}w\_{ih}^{k}x\_{i}^{k}$ , h = 1,……,q (3)

δ($ z\_{h}^{k}$) = 1/(1+$e^{-z\_{h}^{k}}$) , h=1,……,q (4)

δ ($z\_{0}^{k}$) = 1,

Where $w\_{oh}^{k}$ are the biases of the hidden neurons, and δ ($z\_{0}^{k}$) is the hidden layer bias neuron signal which is independent of the iteration index.

The output layer neuronal activations for the backpropagation will be

 $y\_{j}^{k}$ = $\sum\_{h=0}^{q}w\_{hj}^{k}$ δ ($ z\_{h}^{k}$), j=1……, p (5)

.δ$(y\_{j}^{k}$) = 1/ (1+$e^{-y\_{j}^{k}}$), j=1,…..,p (6)

Where $w\_{oj}^{k}$are the biases of the output neurons.

The learning rate η in the Backpropagation algorithm has to be kept small in order to maintain a smooth trajectory in weight space, because large learning rate can lead to oscillations during learning.

 $Δw\_{hj}^{k}$ = η$\sum\_{t=1}^{k}α^{k-t}$ $δ\_{j}^{t}s\_{h}^{t}$ = -η$\sum\_{t=1}^{k}α^{k-t}$ $\frac{∂ε\_{t}}{∂w\_{hj}^{t}}$ (7)

The above equation generalizes the weight change at the *k* th iteration in terms of the weight gradient at each of the previous iterations (Satish Kumar, 2007).

 The computations in the hidden layer is performed by the input training pattern (Yegnanarayana, 2005). The function approximation interpretation of a single layer feedforward neural network enables us to view different hidden layers of the network performing different functions.

**Fuzzy Logic:**

 Fuzzy logic arose out of a fundamentally different way of dealing with uncertainty. Zadeh (1965) introduced a theory of mathematical objects called fuzzy sets — sets wherein the boundaries are not precise. Fuzzy logic (based on the mathematical manipulation of fuzzy sets) provides another approach towards modelling complex systems, a different approach based on a common sense, intuition and natural language, where precise mathematical formulations of chemical and physical components of a system are replaced by natural linguistic rules based on expert human understanding of the natural system. The system of non-linear features can be explained by fuzzy logic with some precise knowledge-based approach. There is a wealth of observational and experimental data in the geological and geophysical sciences. Moreover, a complicated mix of quantitative and qualitative data types are available which are necessary to characterize the non-linear geophysical system. For example, in a coastal aquifer, the shallow depth system may not be linear due to the complex composition of soils. This needs an efficient tool to map the shallow depth system. Many hundreds of samples with tremendous accuracy and repeatability may provide a statistical result. However, these samples are usually scattered over many hundreds or even thousands of kilometres and are arbitrarily located. The output definitely brings out vagueness in the result while interpreting. This kind of vagueness can be rectified by the fuzzy logic tool, since its output is based on an efficient knowledge-based platform.

The primary purpose of fuzzy logic is to formalize reasoning in natural languages. This requires that propositions expressed in natural language be properly formalized. In fuzzy logic, the various components of natural language propositions (predicates, logical connectives, truth qualifiers, quantifiers, linguistic hedges etc.) are represented by appropriate fuzzy sets and operations on fuzzy sets (Zadeh, 1975). Each of these fuzzy sets and operations is strongly context-dependent and consequently, must be determined in the context of each application (Klir & Yuan, 1995). The distinct advantages in using the fuzzy logic approach are to characterize geophysical methods rather than empirical equations. First, fuzzy sets describe systems in natural languages. More importantly, the shapes of the membership functions can easily be changed by small increments, thereby allowing rapid “sensitivity analysis” of the effects of changing the boundaries of the fuzzy sets. When the shapes and boundaries of the membership functions are slightly changed, the output function is also slightly changed. The output values of a fuzzy inference system are changed in order to match more nearly the ground truth.

Rantitsch (2000) used fuzzy *c*-means clustering of elements measured in stream samples from the Alps of Austria to establish four categories of background levels of various elements. These categories were better able to screen out non-anomalous concentrations of metals. Pongracz et al. (1999) tried to develop a drought prediction model for the Great Plains of the US to a long-term record of droughts in eight regions of Nebraska. Stehlik and Bardossy (2002) extended this approach to the general stochastic prediction of a time series for precipitation over Europe. They developed a fuzzy classification of point measurements of geopotential atmospheric pressure surfaces over a large-scale grid of Europe as an input into their model.

  **a) Membership functions**

Let *X* be a space of objects and *x* be an element of *X.* A classical set *A, A⊆X, is* defined as a collection of elements or objects$ x\in X$, such that each *x* can either belong or not belong to the set *A*. By defining a characteristic functionfor each element *x* in *X*, a classical set A can be represented by a set of ordered pairs (*x, 0*) or (*x*, 1), which indicates *x*$ \notin A$or *x* $\in A, $respectively. If *X* is a collection of objects denoted generically by *x,* then a fuzzy set *A* in *X* is defined as a set of ordered pairs:

 A =$\left\{x \in X\right\}$ …. (3.2)

where $µ\_{A}\left(x\right)$ is called the membership function (MF) for the fuzzy set *A*. The MF maps each element of *X* to a membership grade (or membership value) between 0 and 1 (normalizing function is applied here to logically describe the input patterns). In this fuzzy geophysical approach, this can be attained by normalizing the input data values to map the output parameters that are distributed in the random space.

 Obviously, the definition of a fuzzy set is a simple extension of the definition of a classical set in which the characteristic function is permitted to have any values between 0 and 1. If the value of the membership function $µ\_{A}\left(x\right) $is restricted to either 0 or 1, then *A* is reduced to a classical set and $µ\_{A}\left(x\right)$ is the characteristic function of *A.* For clarity, classical sets may be referred to as ordinary sets, crisp sets, non-fuzzy sets, or just sets.

 Usually, *X* is referred to as the universe of discourse, or simply the universe, and it may consist of discrete (ordered or non-ordered) objects of continuous space. The universe of discourse or universal set is the set which, with reference to a particular context, contains all possible elements having the same characteristics and from which sets can be formed. For example, the universal set of all students in a university.

A fuzzy set is completely characterized by itsMF. Since most fuzzy sets in use have a universe of discourse ***X*** consisting of the real line ***R,***it should be impractical to list all the pairs defining a membership function. A more convenient and concise way to define an MF is to express it as a mathematical formula. The derivatives of MFs are important for fine-tuning a fuzzy inference system to achieve a desired input/output mapping. In the Fuzzy Inference System (FIS) training, input clusters were mapped with the membership functions (Jang, 1993).

The AB/2 and ρa values are normalized (so as to obtain a membership grade between 0 and 1)which is required for clustering analysis. Fuzzy subtractive clustering has been applied here in this approach. The number of clusters framed is proportional to the number of rules of the membership functions, and the synthetic data is generated based on this membership function. Thus the input/output mapping has been done with this approach. Stanley Raj et al. 2016 applied fuzzy logic based forecast model for groundwater vulnerability studies.

The membership function used in the primary class training of the fuzzy geo-electrical inversion algorithm as input membership function is ‘gaussmf’ and the output membership function type is 'linear'. Some of the significant membership functions have been discussed here.

**3.2.1. TRIANGULAR MEMBERSHIP FUNCTIONS**

A **triangular MF** is specified by three parameters {a, b, c} as follows:

 triangle (*x; a, b ,c*) = $\left\{\begin{array}{c}0, x\leq a\\\frac{x-a}{b-a}, a\leq x\leq b.\\\frac{c-x}{c-b}, b\leq x\leq c.\\0, c\leq x.\end{array}\right.$ ….(3.3)

By using min. and max. an alternative expression for the preceding equation may be expressed as follows

 triangle (*x; a, b, c*) = $max⁡(min\left(\frac{x-a}{b-a},\frac{c-x}{c-b}\right),0$) …. (3.4)

The parameters {a, b, c} (with a<b<c) determine the *x* coordinates of the three corners of the underlying triangular MF. Figure 3.2 (a) illustrates the triangular membership function

**3.2.2. TRAPEZOIDAL MEMBERSHIP FUNCTIONS**

A **trapezoidal MF** is specified by four parameters {*a, b, c, d*} as follows:

 trapezoid (*x; a, b, c, d)* = $\left\{ \begin{array}{c}0, x\leq a\\\frac{x-a}{b-a}, a\leq x\leq b.\\ 1, b\leq x\leq c.\\\frac{d-x}{d-c}, c\leq x\leq d.\\0, d\leq x.\end{array}\right.$ …. (3.5)

An alternative concise expression using min and max is

 trapezoid (*x; a, b ,c, d*) = $max⁡(min\left(\frac{x-a}{b-a},1,\frac{c-x}{c-b}\right),0$) …. (3.6)

The parameters {*a, b, c, d*} (with a < $b\leq c<d$) determine the *x* coordinates of the four corners of the underlying trapezoidal MF. Figure 3.2 (b) illustrates the trapezoidal membership function.

 Due to their simple formulas and computational efficiency, both triangular MFs and trapezoidal MFs have been used extensively, especially in real-time implementations. However, since the MFs are composed of straight line segments, they are not smooth at the corner points specified by the parameters. The following are some types of MFs defined by smooth and non-linear functions which are more applicable in solving non-linear problems.

**3.2.3 GAUSSIAN MEMBERSHIP FUNCTIONS**

A **Gaussian MF** is specified by two parameters {c, σ}:

gaussian (*x: c,* σ) = $e^{- \frac{1}{2} \left(\frac{x-c}{σ}\right)^{2}}$ …. (3.7)

A Gaussian MF is determined completely by *c* and ; *c* represents the MF’s center and determines the MF’s width. Figure 3.2 (c) illustrates the gaussian membership function

**3.2.4. GENERALIZED BELL SHAPED MEMBERSHIP FUNCTIONS**

A **generalized bell MF** (or **bell MF**) is specified by three parameters {a, b, c}:

bell (*x; a, b, c*) = $\frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$ …. (3.8)

where the parameter *b* is usually positive. (If *b* is negative, the shape of this MF becomes an upside-down bell). Figure 3.2 (d) illustrates the bell shaped membership function. Because of their smoothness and concise notation, Gaussian and bell MFs are becoming increasingly popular for specifying fuzzy sets. Gaussian functions are well known in probability and statistics, and they possess useful properties such as invariance under multiplication (The product of two Gaussians is a Gaussian with a scaling factor) and Fourier transform (The Fourier transform of a Gaussian is still a Gaussian) (Jang, 1993). The appropriate scaling factor between the non-linear mapping of AB/2 and apparent resistivity values in geophysical applications can be done using this Gaussian MF.

**INFERENCE MECHANISM**

Inference mechanism allows mapping between the given input to output parameters using fuzzy logic. The most common types are Mamdani and Sugeno inference systems and they vary in ways of determining outputs. The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator are exactly the same for both the systems. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant (MATLAB R2008b).

The fuzzy inference system is a popular computing framework based on the concepts of fuzzy set theory, fuzzy if-then rules and fuzzy reasoning. It has found successful applications in a wide variety of fields, such as automatic control, data classification, decision analysis, expert systems, time series prediction, robotics and pattern recognition. Because of its multidisciplinary nature, the fuzzy inference system is known by numerous other names, such as fuzzy-rule-based system, fuzzy expert system (Kandel, 1992), fuzzy model (Takagi and Sugeno, 1985), fuzzy associative memory (Kosko, 1992), fuzzy sets on groundwater risk management (Lee, 1994, 1995) and simply (and ambiguously) fuzzy system.

 The basic structure of a fuzzy inference system consists of three conceptual components: a rule base, which contains a selection of fuzzy rules based on the cluster centers; a database (or dictionary), which defines the membership functions used in the fuzzy rules; and a reasoning mechanism, which performs the inference procedure upon the rules and given facts to derive a reasonable crisp output or conclusion.

 Note that the basic fuzzy inference system can take either fuzzy inputs of crisp inputs, but the outputs it produces are almost always fuzzy sets. Sometimes it is necessary to have a crisp output, especially in a situation where a fuzzy inference system is used as a controller or non linear system estimation. Therefore, a method of defuzzification is needed to extract a crisp value that best represents a fuzzy set. A fuzzy inference system with a crisp output is shown in Figure 3.4, where the dashed line indicates a basic fuzzy inference system with fuzzy output and the defuzzification block serves the purpose of transforming an output fuzzy set into a crisp single value. Figure 3.4 shows the rules framed after the process clustering is completed. These rules correspond to the membership functions for fixing the parameters of FIS. The weighted average defuzzification method has been used in this approach for estimating the subsurface layer parameters.

 Thus a fuzzy inference system, with crisp inputs and outputs, implements a non-linear mapping from its input space to the output space. This mapping is accomplished by a number of fuzzy if-then rules that are automatically generated while performing the membership function grades. Each rule will describe the local behavior of the mapping. In particular, the antecedent of a rule defines a fuzzy region in the input space, while the consequent specifies the output in the fuzzy region.

**DEFUZZIFICATION INTERFACE**

It performs the following functions

A scale mapping which converts the range of output values into the corresponding universe of discourse. Thus in this case, apparent resistivity data on a certain range with same characteristics forms a universe of discourse, in order to map the input- output parameters. Defuzzification is nothing but the conversion of fuzzy values of crisp values. Following are some of the defuzzification methods discussed here. Each method has its own advantages.None of them has proved their advantage over the others. Now-a-days the choice of the defuzzification procedure is based mainly on personal preference. The first two methods produce an integral output considering all the elements of the resulting fuzzy set with the corresponding weights. The other methods take into account just the elements corresponding to the maximum points of the resulting membership functions. Lee (1990b) states that*,*unfortunately, there is no systematic procedure for choosing a defuzzification strategy. Although the process of reducing the final fuzzy set to a crisp value does seem appropriate for non-linear problems, much information is lost by doing this, and further work needs to be done on how to use the information available in the solution fuzzy set.

***(a) Centre of area/Gravity defuzzification***

The centre-of-area method is the most well known defuzzification method. It is also known as centre of gravity method or centroid method. It obtains the centre of area ($x^{\*}$) occupied by the fuzzy set. It is given by the expression
 $x^{\*}=\frac{\sum\_{i=1}^{k}x\_{i}μ(x\_{i})}{\sum\_{i=1}^{k} μ(x\_{i})}$ …. (3.12)

In the continuous case we obtain
 $x^{\*}=\frac{∫\_{x}xμ\left(x\right)dx}{∫\_{x} μ\left(x\right)dx}$ …. (3.13)

where is the classical integral,$ x\_{i}$’s are the elements and $μ\left(x\right)$ is the combined membership function. This method determines the center of the area below the combined membership function. Defuzzification refers to the way a crisp value is extracted from a fuzzy set as a representative value.

***(b) Centre of sums method (COS)***

In the centroid method, the overlapping area is counted once whereas in centre of sums, the overlapping area is counted twice. COS builds the resultant membership function by taking the algebraic sum of outputs from each of the contributing fuzzy sets. The defuzzified value $ x^{\*}$ is given by

 $x^{\*}$= $\frac{\sum\_{i=1}^{N}x\_{i}\sum\_{k=1}^{n}μ\_{A\_{k}}(x\_{i})}{\sum\_{i=1}^{N}\sum\_{k=1}^{n}μ\_{A\_{k}}(x\_{i})}$ …. (3.14)

Here, n is the number of fuzzy sets and N the number of fuzzy variables. COS is actually the most commonly used defuzzification method.

***(c) Mean of maximum MOM***

MOM is the simple way of defuzzying the output to take a crisp value with the highest degree of membership. In cases with more than one element having the maximum value, the mean value of the maxima is taken. The equation of the defuzzified value$ x^{\*}$ is given by

 $ x^{\*}$ = $\frac{\sum\_{x\_{i}\in M}^{}x\_{i}}{\left|M\right|}$ …. (3.15)

where M = {$x\_{i} μ\left(x\_{i}\right)$ is equal to the height of fuzzy set}

#### (d) Weighted Average Defuzzification Technique

In this method, the output is obtained by the weighted average of each output of the set of rules stored in the knowledge base of the system.

The weighted average defuzzification technique can be expressed as

$ x^{\* }= \frac{\sum\_{i=1}^{n}m^{i}w\_{i}}{\sum\_{i=1}^{n}m^{i}}$ …. (3.16)

where *x\** is the defuzzified output, *mi* is the membership of the output of each rule, and *wi* is the weight associated with each rule. This method is computationally faster and easier, and gives fairly accurate results. This defuzzification technique is applied in the fuzzy logic-based geoelectrical inversion algorithm.



**ANFIS Architecture**

ANFIS is a combination of logical fuzzy systems and neural networks. This kind of inference system has the adaptive nature to rely on the situation it trained. Thus it has lot of advantages from learning to validating the output. Takagi-Sugeno fuzzy model is shown in the Fig 4.

            As shown in Fig 2, the ANFIS system consists of 5 layers, layer symbolized by the box is a layer that is adaptive. Meanwhile, symbolized by the circle is fixed. Each output of each layer is symbolized by *O 1, i* with i is a sequence of nodes and l is the sequence showing the lining. Here is an explanation for each layer, namely:

**Layer 1.**

Serves to raise the degree of membership

*O1,i = μA(x)* i=1,2 (8)

and

*O1,i = μB(y)* i=1,2 (9)

with x and y are the input for the i-th node

*μA(x) =* 1/[1+(det(x-ci)/ai)^2bi](10)

by {ai , biand ci} are the parameters of membership function or called as a parameter *premise* .

 **Layer 2**

Serves to evoke *firing-strength* by multiplying each input signal.

*O2,i = wi = μA(x)* x *μB(y) i=1,2* (11)

  **Layer 3**

Normalize the *firing strength*

*O3,i =*$\overline{w\_{i}}$= $\frac{w\_{i}}{w\_{1}+w\_{2}}$ i=1,2 (12)

**Layer 4**

  The values of the premise parameters are fixed as per the layer 1, the overall output can be expressed as a linear combination of the consequent parameters. Thus the consequent parameters are not fixed in the forward pass but it is kept fixed while backward pass when the sytem seeks the weight updation. Hybrid learning procedure for ANFIS. ). Srinivas et al., 2012 applied Neuro fuzzy technique for inverting geoelectrical resistivity data.

|  |  |  |
| --- | --- | --- |
|  | **Forward Pass** | **Backward Pass** |
| **Premise parameters** | Fixed | Gradient descent |
| **Consequent parameters** | Least-squares estimator | Fixed |
| **Signals** | Node outputs | Error signals |

         Calculating the output based on the parameters of the rule *consequent* {pi , qi and ri}

*O4,i =*$\overline{w\_{i}}f\_{i}$ *=* $\overline{w\_{i}}$ *(pix+qiy+ri)* (13)

**Layer 5**

Counting the ANFIS output signal by summing all incoming signals will produce

$ \sum\_{i}^{}\overline{w\_{i}}f\_{i}$= $\frac{\sum\_{i}^{}w\_{i}f\_{i}}{\sum\_{i}^{}w\_{i}}$ (14)

**Discussions:**

Geophysical prospecting attempts to solve ubiquitous inverse problems using soft computing approach. Model parameters can be obtained using specific computational aspects of inverse theory gain importance in geophysical processing technology. The theoretical and computational approach is based on the iterative algorithm to fit model parameters appropriate to real world model. Geophysical inversion algorithm works well if the algorithm is semi supervised so that the user can change the model parameters to adapt the system trained model. Recently 3d inversion algorithms are more popular in investigating the subsurface parameters, mineral. Petroleum or oil exploration.

**References:**

[1] Ahmad Neyamadpour W.A., WanAbdullah T., SamsudinTaib 2010 Inversion of

Quasi-3D DC resistivity imaging data using artiﬁcial neural networks. *Journal of Earth System Science*, **119**, 27–40

[2] Brown M., Poulton M. 1996 Locating buried objects for environmental site

investigations using neural networks. *Journal of Environmental and Engineering Geophysics*, 1, 179-188

[3] Christensen NB, Sørensen KI (1998): Surface and borehole electric and

electromagnetic methods for hydrogeological investigations. –European Journal of Environmental and Engineering Geophysics **3**: 75–90.

[4] Constable S., Parker R., Constable C. 1987 Occam's inversion: a practical algorithm for generating smooth models from electromagnetic sounding data. *Geophysics*, **52**, 289-300

[5] Dey A., Morrison H. 1979 Resistivity modeling for arbitrarily shaped two-dimensional Structures. *Geophysical Prospecting*,**27**, 106-136

[6] Flathe H 1955 A practical method of calculating geoelectrical model graphs for horizontally stratified media. *Geophysical Prospecting*, **3**, 268-294

[7] Gad El-Qady and Keisuke Ushijima 2001 Inversion of DC resistivity data using neural networks. *Geophysics Prospecting*, **49**, 417-430

[8] Ghosh DP (1971) Inverse filter coefficients for the computation of the apparent resistivity standard curves for horizontally stratified earth. *Geophysical Prospecting*, **19**, 769-775

[9] Griffith D., Barker R 1993 Two-dimensional resistivity imaging and modeling in areas of complex geology. *Journal of Applied Geophysics*, **29**, 211-226

 [10] Jang J S R (1993) Adaptive-network based fuzzy inference system*.* IEEE Trans.

 Systems, Man and Cybernetics 23: 665-685

[11] Jacek M Zurada 2006 *Introduction to Artificial Neural Systems*, Jaico Publishing

 house, Mumbai.

[12] Jimmy Stephen C., Manoj Singh S.B. 2004 A direct inversion scheme for deep resistivity sounding data using artificial neural networks. *Proceedings of Indian Academy of Sciences.(Earth and Planetary Science)*, **113**, 49-66

[13] Kandel, A. [1982], *Fuzzy Techniques in Pattern Recognition*. JohnWiley, New York.

[14] Klir, G. J., &Yuan, B. [1995], *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Prentice-Hall, Upper Saddle River, NJ.

 [15] Kosinky W.K., Kelly W.E. 1981 Geoelectrical sounding for predicting aquifer properties*. Groundwater*,**19**,163-171

 [16] Kosko, B. [1992], *Neural Networks and Fuzzy Systems*. Prentice-Hall, Englewood Cliffs, NJ.

[17] Lee,Y.W., Dahab, M. F., &Bogardi, I. [1994], “Fuzzy decision making in ground water nitrate risk management.” *Water Resources Bulletin*, **30**(1), 135–148.

[18] Lee, Y. W., Dahab, M. F., Bogardi, I. [1995], “Nitrate risk assessment using a fuzzy-set approach.” *Journal of Environmental Engineering*, **121**(3), 245–256.

[19] Loke M., Barker R. 1996 Rapid least-squares inversion of apparent resistivity Pseudo- sections using a quasi-Newton method. *Geophysical Prospecting*,**44**, 131-152

[20] Mazac O., Kelly W.E., Landa I. 1985 A hydrophysical model for relation between electrical and hydraulic properties of aquifers. *Journal of Hydrology*, **79**, 1-19

[21] Pongracz, R., Bogardi, I., & Duckstein, L. [1999], Application of fuzzy rule-based modeling technique to regional drought. *Journal of Hydrology*, **224**(3–4), 100–114.

[22] Rantitsch, G. [2000], “Application of fuzzy clusters to quantify lithological background concentrations in stream-sediment geochemistry.” *Journal of Geochemical Exploration*, **71**(1), 73–82.

[23] Satish kumar 2007 *Neural networks - A class room approach*. Tata McGraw-Hill Publishing Limited, New Delhi.

[24] Smith N.,Vozoff K. 1984 Two-dimensional DC resistivity inversion for dipole-dipole data. *IEEE Transactions on Geoscience and Remote Sensing*, **22**, 21-28

[25] Sørensen KI, Auken E, Christensen NB, Pellerin L (2005): An Integrated Approach for Hydrogeophysical Investigations: New Technologies and a Case History. – In Butler D K (ed.) Near-Surface Geophysics **2**,Investigations in Geophysics **13**: 585–603. Society of Exploration Geophysics.

[26] Sreekanth P.D., Geethanjali N., SreeDevi, Shakeel Ahmed P.D., Ravikumar N., Kamala Jayanthi P.D. 2009 Forecasting Groundwater level using artificial Neural networks. *Current Science*, **96**, 933-939

[27] Srinivas Y., Stanley Raj A., Hudson Oliver D., Muthuraj D., and Chandrasekar N (2012). Estimation of subsurface strata of earth using Adaptive Neuro-Fuzzy Inference System (ANFIS). **Acta Geod. Geoph. Hung., (Springer Publications) 47(1), 78–89.**

[28] Stanley Raj.A., Y.Srinivas, D.Hudson Oliver D. Muthuraj, (2014) A novel and generalized approach in the inversion of geoelectrical resistivity data using Artificial Neural Networks (ANN). **Journal of Earth System Sciences 123 (2), 395–411.**

[29] A.Stanley Raj**,** D.Hudson Oliver, Y.Srinivas, “Forecasting groundwater vulnerability in coastal region of southern Tamil Nadu – a Fuzzy based approach” **Arabian Journal of Geosciences**  (2016), 9:351.

[30] Stehlik, J., & Bardossy, A. [2002], “Multivariate stochastic downscaling model for generating daily precipitation series based on atmospheric circulation.” *Journal of Hydrology*, **256**(1–2), 120–141.

[31] Takagi, T., & Sugeno, H. [1985], “Fuzzy identification of systems and its application for modeling and control.” *IEEE Transactions on Systems, Man and Cybernetics*, **15**(1), 116–132.

[32] Tripp A., Hohmann G., Swift C. 1984 Two-dimensional resistivity inversion. *Geophysics*,**49**, 1708-1717

[33] Yadav G.S., Abolfazli H. 1998 Geoelectrical soundings and their relationships to

hydraulic parameters in semi arid regions of Jalore, North West India. *Journal of Applied Geophysics*,**39**, 35-51.

[34] Yegnanarayana B. 2005 *Artificial Neural Networks*. Prentice Hall of Indi, Private

 Limited, New Delhi.

 [35] Zadeh L. 1965 *Fuzzy sets* Inf Control **8**: 338-353

http://www.geol-amu.org/notes/b8-4-4.htm