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A Survey of certain Mathematical Applications in Theoretical Physics

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Abstract - Mathematics is a wonderful subject and an essential tool in physics, especially in theoretical physics, which relies heavily on mathematics (by which we mean here mainly Field Theory and High Energy Physics). Quantum Mechanics and Special Theory of Relativity are the foundations of these fields of physics, and numerous mathematical concepts are used in them. Differential geometry, infinite series, Mellin transforms, Fourier and integral transforms, special functions, calculus, complex algebra, topology, group theory, Riemannian geometry, functional analysis, linear algebra, and operator algebra are just a few of the topics we'll cover in this paper. Mathematical tools utilized in physics will also be discussed in this course. It is not incorrect to state that mathematics is a powerful tool, without which there would be no Physics theory!! In addition, a brief summary of our research is given.

Keywords - Certain Mathematical Applications, Theoretical Physics, Quantum Mechanics, Field Theory, High Energy Physics, Physics theory

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INTRODUCTION

Physics cannot function without mathematics, which is both a beautiful subject and a necessary and potent tool. As far as I'm concerned, there is no physics without mathematics. Here, we'll focus on only a few examples of how mathematics is used in theoretical physics (TP). PP, QFT, and Theoretical High Energy Physics (THEP) are all examples of theoretical physics (THEP). PP, QFT, and THEP can be used to address questions about the nature of matter, its constituents, and how they interact with each other. It is the atoms in molecules that are made up of nucleons, which in turn are formed of quarks and gluons, which make up the nucleons. Quarks and leptons are two types of basic particles that make up most of the stuff in the universe. For elementary particles, gauge bosons mediate three types of forces and the Higgs Boson is assumed to give them mass, via interactions with them. The interaction of subatomic particles can be explained in part by the Quantum Field Theory. A particle's existence can be shown by bringing another particle into its neighborhood, and this is known as the presence of the first particle's Field. There are an endless number of ways in which a field can be defined. Particles in QFT can be created or destroyed by quantizing the fields associated with them. According to Heisenberg's Uncertainty principle, if we want to investigate the structure of the nucleus, the probe must have a very high energy. The probe should have a wavelength of the order of size of nucleus

$$\lambda \sim 10^{-15} m \qquad (1)$$

and Heisenberg's principle says that

$$\lambda \sim \frac{\hbar}{p} \qquad (2)$$

where p = mv is the linear momentum of the probe (particle). For example, when a proton moves at the speed of ~ 0.85 c (where c = 3 ×108 m/s is the velocity of light) it will have wavelength λp

$$\lambda_p \sim 1.55 \times 10^{-15} meters \qquad (3)$$

When an electron of mass 9.1×10−31 kg travels at the speed of 5.3×106m/s, it will have a Kinetic Energy (K.E) ~ 1.284 ×10−17 Joules, and wavelength of the order of 0.137 nanometer. (1 nanometer = 10−9 m). In PP and Nuclear Physics, a convenient unit of energy is defined as electron volt (eV), where 1 eV is the energy gained by an electron when accelerated through a potential difference of 1 Volt.

$$1eV = 1.6 \times 10^{-19} Joules \qquad (4)$$

Energy consumed by a bulb of 100 watt in one second = 6.2 ×1024 eV, and when 2 electrons are annihilated, energy ~ 1 MeV is produced (1 MeV = 106 eV). Mass of one hydrogen atom ≈ 931 MeV (1 TeV = 1012 eV). 1 TeV = 1012 eV is just about the K.E of a flying mosquito. Our daily lives have a much larger energy budget than the energies of elementary particles traveling at ultra-relativistic speeds. Because of their little mass, they are referred to as high-energy objects on the subatomic scale. The interaction of fundamental (elementary) particles and how they acquire mass are both clarified by QFT and Gauge theories. Mathematical techniques utilized in theoretical physics will be discussed in the following paragraphs. To be clear, the list of strategies described here is not comprehensive; there are many more, and it would be impossible to include them all here. Some examples of how the above-mentioned Mathematical approaches are utilized in physics follow.

Group Theory

There exist conserved quantities in a physical system if a Lagrangian L is invariant under specific symmetry transformations. According to Noether's Theorem, symmetry transformations play a significant role in physics. If L is invariant during translation in space, then the linear momentum is conserved, to use the most basic of examples. Groups can be formed as a result of such transitions. So-called "rotation groups" are formed (n). Invariance under the U(1) group, known as gauge tarnsformations, results in the conservation of electric charge. Scientists discovered that the nuclear force is invariant under group SU(2), where the fundamental representations are two-dimensional, because of the symmetry of the nuclear force under neutron-proton exchange (np) (a doublet).

$$\begin{pmatrix} n \\ p \end{pmatrix} \tag{5}$$

Similarly the three quarks form a fundamental representation of SU(3)

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} \tag{6}$$

Or,

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{7}$$

Here R, G, B stand for red, green and blue quarks, and u, d, s are up, down, strange type of flavors of quarks. Accordingly, we have gauge theories for the respective groups U(1)Y , S U(2)L and S U(3)c. Gauge theories help explain the intearctions among fundamental particles. Also, elementary particles can be grouped according to these groups. Each group has a generator, and they obey lie algebra. For example, the three Pauli matrices σ1, σ2, σ3 are generators of the group SU(2):

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ 0 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{8}$$

and lie algebra is

$$\{\sigma_j, \sigma_k\} = 2i\epsilon_{jkl}\sigma_l \tag{9}$$

Where jkl is totally antisymmetric tensor of rank 3.

Calculus

Infact, calculus is used so frequently in Physics − in all equations of motions, to find trajectory of the particles in solving field equations, almost everywhere differentiation and integration is used. Linear momentum in Quantum Mechanics is defined as

$$p = -i\hbar\frac{\partial}{\partial x} \tag{10}$$

and Hamiltonian can be expressed in terms of p. Integration, differentiation, integral and differential equation form the backbone of Theoretical Physics. Quantum Mechanics when combined with the concepts of STR (Special Theory Of Relativity), yields the Quantum Field Theory

Tensors and matrices

Tensors are used, e.g in STR in which the invariant interval between two physical events can be expressed as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

wheregμν is the metric tensor, and dx is the 4-dimensional spacing between two points. In Minkowski's space

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{11}$$

Moreover, in the QFT, all field equations are used in 4-vector notation (Einstein's unified 4-dimensional space), with Lorentz covariant formulations. Hence tensors form necessary ingredients for THEP, e.g, the Dirac field equation is,

$$(i\gamma_\mu\partial_\mu + m)\psi = 0 \tag{12}$$

Also, in QM, wave functions are taken as components of a Hilbert space and are treated as column/row vectors. Operators are treated as matrices in Matrix Mechanics formulation of Q.M. For example, LZ, the third component of angular momentum operator is

$$L_Z = i\frac{h}{2\pi}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{13}$$

Complex Analysis

One of the important applications of Complex Analysis in THEP is in Feynman's propagator, i.e, the function that explains the propagation of a particle from one point in space to another. It can be expressed in terms of Green's function

$$G(r, r') = \frac{1}{(2\pi)^4}\int d^4p \frac{e^{ip.(r-r')}}{p^2 - (k^2 + i.\epsilon)} \tag{14}$$

Along the semicircle, the complex-p plane integration is performed. When calculating the cross-section of all physical processes in QFT and HEP, the Feynman propagator provides the basis of Feynman rules, which enable to compute the contributions of all scattering and interaction processes.

Differential and Riemannian Geometry

It is a discipline of mathematics that use differential calculus, integral calculus, and linear algebra to investigate geometry. Differential topology and Riemannian geometry are two subfields of this larger field. General Theory of Relativity (GTR) relies heavily on the Lagrangian-Hamiltonian formulations of PP and QFT (GTR). Differential geometry's branch on Riemannian Manifolds is called Riemannian geometry. A manifold is a space that can be parameterized by a set of real parameters that change over time (called local co-ordinates). Because of the smooth transitions in the inner products on the tangent space at different points, this is Riemannian space. Angle, length of curves, surface area, volume, etc. may all be measured using this method. String Theory and GTR both heavily rely on it. As a result, it may be used to describe how geodesics behave on surfaces, which is useful for understanding manifolds in higher dimensions.

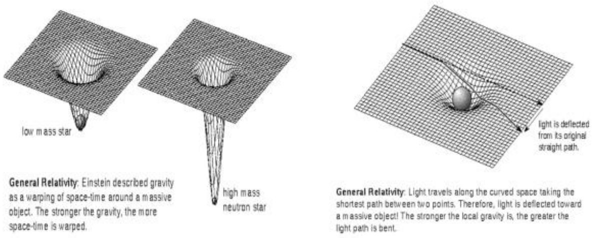

Figure 1: Einstein's space time curvature

CONCLUSION

The interaction of subatomic particles can be explained in part by the Quantum Field Theory. If a particle's existence can be confirmed by bringing another particle into its proximity, by the action of a force between them, we refer to this as the Field of the first particle. There are an endless number of ways in which a field can be defined. Field quantization is used in QFT to produce or destroy particles, and each particle is associated with a certain field. Among the topics we've studied include grand unified theories, neutrino physics, fermion masses, and mixing. Nuclei that are so little that they don't interact with other fundamental particles are called neutrinos. They come in three varieties and mix with each other while on the road. Cosmic rays and Earth-based rays are the sources of these rays. However, gravity is not included in Grand Unified Theories because it is the only force that does not exist in the universe.

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