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String Cosmology in Bulk Viscous Bianchi Type – III Space-Time

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Abstract – The present paper provides solution of Bianchi type- III bulk viscous string cosmological model of assuming that (i) $\xi^2 = \mu^2\theta$ and (ii) $\sigma \propto \theta$ which leads to $\beta = \gamma^\zeta$ (where μ and ζ are constants, ζ is bulk viscosity and θ is scalar of expansion). Various physical and geometrical features of the model are also found and discussed.

Key Words – Cosmological Model, String Cosmology, Viscosity, Shear, Expansion, Metric Potential.

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1. INTRODUCTION:

The general relativistic treatment of strings was initiated by Letelier [7] and Stachel [17]. This model has been used as a source for Bianchi type-I and Kantowski-Sachs cosmologies by Letelier [7]. After wards, Krori et. al. [16, 6(a)] and Wang [21, 22, 23] have discussed the solutions of Bianchi type-II, VI, VIII and IX for a cloud string. Tikekar and Patel [18], Chackraborty and Chackraborty [4, 5] Singh et. al. [16], Turyshev [19] and Vilenkein [20] have presented the exact solutions of Bianchi type – III and sphericaly symmetric cosmology respectively for a cloud string.

Recently bali andf Dave [2(a)] have presented Bianchi type-III string cosmological model with bulk viscosity, where the constant coefficient of bulk viscosity is considered. However, it is known that the coefficient of bulk viscosity is not constant but decreases as the universe expands [2]. Arbab [1], Pradhan et. al. [8-10], Ray and Mukhopadhay [11], Singh and Singh [12], Singh and Pradhan [13], Singh and Kumar [14,15], Yadav et. al. [25, 26] Yadav and Kumar [27] are some of the authors who have studied various aspects of interacting fields in the framework of Bianchi type-III string cosmological model with bulk viscosity.

In this chapter, we have investigated Bianchi type-III bulk viscous string cosmological model. To obtain a determinate solution, we have assumed that $\xi^2 = \mu^2\theta$ (where ξ is bulk viscosity, θ is scalar of expansion and μ is a constant) and the shear scalar is proportional to scalar of expansion $\sigma \propto \theta$, which leads to the relation between metric $\beta = \gamma^\zeta$. The physical and geometric features of the model are also found and discussed.

2. THE FIELD EQUAITONS

Here we take the Bianchi type-III space-time metric given by [4].

$$(2.1)\quad ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{2x} dy^2 + \gamma^2 dz^2$$

where α, β and λ are the functions of time t alone.

The energy-momentum tensor for a cloud of string with bulk viscosity is [5].

$$(2.2)\quad \tau_{ij} = \rho u_i u_j - \lambda \chi_i \chi_j - \xi\theta(u_i u_j + g_{ij})$$

where $\rho = \rho_p + \lambda$, is the rest energy density of the cloud of strings with particles attached to them, ρ_p is the rest energy density of particles, λ is the tension density of the cloud of string, $\theta = u^i_{;i}$, is the scalar of expansion, and ξ is the coefficient of bulk viscosity. According to Letelier [7] the energy density for the coupled system ρ and ρ_p is restricted to be positive, while the tension density λ may be positive or negative. The vector u^i describes the cloud four-velocity and x^i represents a direction of anisotropy, i.e. the direction of string. They satisfy the standard relations [7].

$$(2.3)\quad u^i u_j = -\chi^i \chi_j = -1,\ u^i \chi_i = 0$$

The expressions for scalar of expansion and shear scalar are (kinematical parameters)

$$(2.4)\quad \theta = u^i_{;i} = \frac{\dot\alpha}{\alpha} + \frac{\dot\beta}{\beta} + \frac{\dot\gamma}{\gamma}$$

$$(2.5)\quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

$$= \frac{1}{3}\left(\frac{\dot{\alpha}^2}{\alpha^2} + \frac{\dot{\beta}^2}{\beta^2} + \frac{\dot{\gamma}^2}{\gamma^2} - \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} - \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}\right)$$

Einstein's equation we consider here is

$$(2.6)\quad R_{ij} - \frac{1}{2}Rg_{ij} - \tau_{ij}$$

where we have choose the units such that c = 1 and $8\pi G = 1$. IN the co-moving coordinates $u^i = \delta^i_0$ and $u^i = -\delta^0_i$, and with the help of Eqs. (2.1) – (2.3), the Einstein equation (2.6) can be written as

$$(2.7)\quad \frac{\ddot{\beta}}{\beta} + \frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} = \xi\theta$$

$$(2.8)\quad \frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} = \xi\theta$$

$$(2.9)\quad \frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\beta}}{\beta} + \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{1}{\alpha^2} = \lambda + \xi\theta$$

$$(2.10)\quad \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} + \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} - \frac{1}{\alpha^2} = \rho$$

$$(2.11)\quad \frac{\dot{\alpha}}{\alpha} - \frac{\dot{\beta}}{\beta} = 0$$

where the dot denotes the differentiation with respect to time t.

3. SOLUTION OF THE FIELD EQUATIONS

From equations (2.11), we obtain

$$(3.1)\quad \alpha = M\beta$$

Now we note that the four independent equation (2.8)-(2.11) in six unknown variables $(\alpha, \beta, \gamma, \lambda, \rho, \xi)$. Thus for complete determinancy of the system we require two more relations connecting these variables.

For this we choose two relations as

$$(3.2)\quad \xi^2 = \mu^2\theta$$

$$(3.3)\quad \beta = \gamma^{\zeta}$$

were ζ is a constant

Substituting Eq. (4.3.3) into Eq. (2.4) and using Eq. (3.2) we have

$$(3.4)\quad \theta = (2\zeta H)\frac{\dot{\gamma}}{\gamma}$$

$$(3.5)\quad \xi\theta - H\left(\frac{\dot{\gamma}}{\gamma}\right)^{3/2}$$

$$(3.6)\quad H = \mu(2\zeta + 1)^{3/2}$$

with the help of equations (4.3.3) and (4.3.5) Eq. (4.2.7) reduces to

$$(3.7)\quad \frac{\ddot{\gamma}}{\gamma} + \frac{\zeta^2}{(\zeta+1)}\frac{\dot{\gamma}^2}{\gamma^2} = \frac{H}{\zeta+1}\left(\frac{\dot{\gamma}}{\gamma}\right)^{3/2}$$

To solve Eq. (3.7), we donote $\dot{\gamma} = A$, then $\ddot{\gamma} = \frac{AdA}{d\gamma}$, and the eq. (3.7) can be cast to the form

$$(3.8)\quad \frac{dA}{d\gamma} + \eta\frac{A}{\gamma} = \frac{H\sqrt{A}}{(\zeta+1)r^{\frac{1}{2}}}$$

where

$$(3.9)\quad \eta = \frac{\zeta^2}{\zeta+1}$$

Equation (3.8) can be written as

$$(3.10)\quad \frac{d}{d\gamma}\left(\sqrt{\eta}^{\frac{1}{2}\eta}\right) = \frac{H^{\mu+2}}{2(\zeta+1)}r^{(2\eta-1)/4}$$

Thus the solution eqn. (3.8) can easily be found as

$$(3.11)\quad A = \left[\frac{H\gamma^{\frac{1}{2}}}{(\zeta+1)(\eta+1)} + C\gamma^{-\frac{\eta}{2}}\right]^2$$

where c is the constant of integration. With the help of Esq. (3.11), the metric (2.1) can be written as

$$(3.12)\quad ds^2 = -\left[\frac{H\gamma^{\frac{1}{2}}}{\zeta^2+\zeta+1} + \gamma^{-\eta/2}\right]d\gamma^2$$

$$+ m^2\gamma^{2\zeta}dx^2 + \gamma^{2\zeta}e^{2x}dy^2 + \gamma^2 dz^2$$

Under suitable transformation of coordinates, Eq. (3.2) reduces to

$$(3.13)\quad ds^2 = -\left[\frac{HT^{\frac{1}{2}}}{\zeta^2+\zeta+1} + CT^{-\eta/2}\right]dT^2$$

$$+ m^2T^{2\zeta}dx^2 + T^{2\zeta}e^{2x}dy^2 + T^2dz^2$$

The expressions for the energy density ρ, the string tension density, the particle density ρ_p, the coefficient of bulk viscosity ξ, the scalar of expression θ and the shear scalar σ^2 for the model (3.13) are given by

$$(3.14)\quad \rho=\zeta(\zeta+2).\left[\frac{H}{\zeta^2+\zeta+1}+CT-\left(\frac{\eta+1}{2}\right)\right]^4-\frac{T-2\zeta}{M^2}$$

$$(3.15)\quad \lambda=\frac{2(\zeta-1)}{h}.\left[\frac{HT^{1/2}}{\zeta+\zeta+1}-\frac{C\eta}{2}T^{-\left(\frac{n+4}{2}\right)}\right]$$

$$+2\zeta(\zeta-1)\left[\frac{H}{\zeta^2+\zeta+1}+CT^{\frac{n+1}{2}}\right]^4-\frac{T^{-(2\zeta)}}{M^2}$$

$$(3.16)\quad \rho_p-2(\zeta-1).\left[\frac{HT^{\frac{1}{2}}}{\zeta^2+\zeta H}+CT^{-\frac{\eta}{2}}\right]^3$$

$$\left[\frac{HT^{-(l+m)}}{2(\zeta^2+\zeta+1)}+\frac{C\eta}{2}T^{\frac{\eta}{2}-1}\right]+\zeta(4-\zeta).\left[\frac{H}{\zeta^2+\zeta+1}+CT^{\frac{\eta+1}{2}}\right]^4$$

$$(3.17)\quad \theta=(2\zeta+1).\left[\frac{H}{(\zeta+\zeta+1)}+CT^{-(\eta+1)}\right]^{-2}$$

$$(3.18)\quad \sigma^2=(\zeta-1)^2.\left[\frac{H}{\zeta^2+\zeta+1}+CT^{-\left(\frac{\eta+1}{2}\right)}\right]^{-4}$$

From Eq. (3.14). It is observed that the standard condition $\rho\geq 0$ is satisfied when

$$(3.19)\quad \zeta(\zeta+2)(\mu+3)\left[\frac{H}{\zeta^2+\zeta+1}+CT^{\frac{n+1}{2}}\right]^4\geq\frac{T^{-2\zeta}}{M^2}$$

It is seen that the scalar of expansion θ tends to infinitely large and the energy density ρ→∞ when T→0, but θ tends to finite and ρ tends to finite when T→∞ due to the presence of bulk viscosity (in the absence of bulk viscosity H = 0, θ→0 and ρ→0 when T→∞). Hence the model represents the shearing and non-rotating expanding universe with the big-bang start.

Therefore the model describes a shearing non rotating expanding universe without the big-bang start. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of universe. Furthermore, since $\text{Lim}_{T\to\infty}\frac{\sigma}{\theta}\neq 0$, the model does not approach isotropy for large values of T. The shear scalar □ is zero when ζ=1, hence ζ=1 is the isotropy condition.

In the absence of bulk viscosity H = 0, the model (3.16) reduces to the string model without viscosity, that is

$$(3.20)\quad ds^2=-C-2T^{2\eta}dT^2+M^2T^{2\zeta}dx^2+T^{2\zeta}e^{2x}dy^2+T^2dz^2$$

$$(3.21)\quad \rho=\zeta(\zeta+2)C^2T^{-2(\eta+1)}-\frac{T^{-2\zeta}}{M^2}$$

$$(3.22)\quad \lambda=\frac{\zeta(\zeta-1)(\zeta+2)}{\zeta+1}C^2T^{-2(\eta+1)}-\frac{T^{-(2\zeta)}}{M^2}$$

$$(3.23)\quad \rho_p=\frac{2\zeta(\zeta+2)}{\zeta+1}C^2T^{-2(\eta+1)}$$

$$(3.24)\quad \theta=(2\zeta+1).CT^{-(\eta+1)}$$

$$(3.25)\quad \sigma^2=(\zeta-1)^2.C^2T^{-2(\eta+1)}$$

From Eq. (3.21) It is observed that the standard condition $\rho\geq 0$ is fulfilled when

$$(3.26)\quad \zeta(\zeta+2).C^2T^{-2(\eta+1)}\geq\frac{T^{-(2\zeta)}}{M^2}$$

The scalar of expansion θ tends to infinitely large when T→0, and θ→0 when T→∞, provide ζ > – ½, and the scalar of expansion in the model is monotonically decreasing when 0<T<∞. Since $\lim_{T\to\infty}\frac{\sigma}{\theta}\neq 0$, the model does not approach isotropy for large values of T. However the energy density ρ→∞ when T→0 and ρ→0 when T→∞, therefore the model describes continuously expanding shearing non-rotating universe with the big-bang start.

4. REMARKS AND CONCLUSION

In this chapter, we have studied the Bianchi type-III string cosmological model with bulk viscosity. To obtain a determinate solution, we assume that the coefficient of the bulk iscosity is a power function of the scalar of expansion for which we have charen $\xi^2 = \mu^2\theta$ and the shear scalar is proportional to scalar of expansion $\delta\propto\theta$, which leads to the relation between metric potential $\beta = \gamma^\zeta$. The physical and geometric features of the model are also discussed. There is a big-bang start in the model.

The scalar of expansion θ is infinitely large at T=0, and θ tends to finite when $T\to\infty$, hence the model represents the shearing and non-rotating expanding universe with the big-bang start. The energy density ρ→∞ when T→0, ρ tends to zero when T→∞. Furthermore, since $\lim_{T\to\infty}\frac{\sigma}{\theta}\neq 0$, the model does not approach isotropy for large values of T. In the absence of bulk viscosity H = 0, this model can reduce to the model (3.20).

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