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Evaluation of Euler's Equation of Motion

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Abstract - In this paper, Euler's equation of motion for steady flow of ideal fluid has been derived in order to solve physical problems related to ideal incompressible fluid in motion. The equation is interesting as it relates the velocity and pressure of fluid with density .The equation being computationally attractive and has applications, one of which is demonstrated through example.

Keywords - Euler's equation of motion, invicid fluid, incompressible, pressure, density

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INTRODUCTION

In this paper we will discuss equation of motion for ideal invicid fluid and its applications. Equation of motion is most fundamental equation in fluid mechanics that uses Newton's 2nd law of motion according to which the rate of change of momentum of particle can be equated to total force applied .We assume that flow is homogenous and incompressible. The flow properties are independent of time and follows continuum hypothesis. The flow is along stream line and velocity of fluid is uniform over cross-section.

We select a portion of fluid having volume δV enclosing material surface δS .Then the rate of variation of linear momentum of selected portion is equated force that acts on that portion of fluid.

Rate of variation of linear momentum in volume V = $\frac{\partial}{\partial t}\int \vec{w}\,\rho\, dV = \int \frac{\overrightarrow{dw}}{dt}\rho\, dV$

Here $\vec{w}$ is velocity of fluid having density.

We know, in general that when we deal with a portion of fluid both surface as well as body forces comes into play.

Total surface force on $\Delta S = -\int p\hat{n}dS$

Then by Gauss divergence theorem, total surface force in volume V becomes = $-\int \nabla p\, dV$

Here p is pressure acting at a point in fluid region. If we denote $\vec{L}$ as body force per unit mass

Then total body force on $\Delta V = \int \rho\, \vec{L}\, dV$

By Newton's second law of motion we have $\int \frac{\overrightarrow{dw}}{dt}\rho\, dV = -\int \nabla p\, dV + \int \rho\, \vec{L}\, dV$

Since δv is arbitrary, we have $\frac{\overrightarrow{dw}}{dt}\rho + \nabla p - \vec{L} = 0$ … (1)

$\frac{\overrightarrow{dw}}{dt}\rho + \nabla p - \vec{L} = 0$ is eulars equation of motion for ideal invicid fluid.

Since $\frac{\overrightarrow{dw}}{dt}$ is total acceleration, therefore $\vec{w}$

$$\frac{\overrightarrow{dw}}{dt} = \frac{\partial \vec{w}}{\partial t} + (\vec{w}\,\nabla)\vec{w} = \frac{\partial \vec{w}}{\partial t} + \nabla(1/2\,\vec{w}^{\,2}) - \vec{w}\times(\nabla\times\vec{w})$$

Thus equation (1) becomes

$$\frac{\partial \vec{w}}{\partial t} + \nabla(1/2\,\vec{w}^{\,2}) - \vec{w}\times(\nabla\times\vec{w}) = \frac{-\nabla p}{\rho} + \vec{L} \quad \ldots (2)$$

If we assume the flow to be of potential (i.e. $\vec{w} = -\nabla\emptyset$ and $\nabla\times\vec{w} = 0$) find under the action of conservative body forces (i.e. $\vec{L} = -\nabla\sigma$) then integration of equation (2) gives well-known equation called Bernoulli's equation of motion.

$$\frac{\partial(-\nabla\emptyset)}{\partial t} + \nabla(1/2\,\vec{w}^{\,2}) = \frac{-\nabla p}{\rho} - \nabla\sigma$$

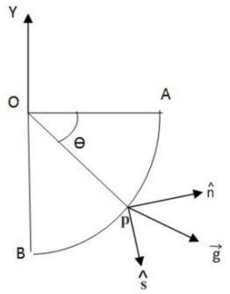
$$-\nabla\frac{\partial\emptyset}{\partial t} + \nabla(1/2\,\vec{w}^{\,2}) + \frac{\nabla p}{\rho} + \nabla\sigma = 0 \quad \ldots (3)$$

Taking dot product with $\overrightarrow{dr}$ equation (3) become

$$\frac{\partial\emptyset}{\partial t} + 1/2\,\vec{w}^{\,2} + \sigma + \int\frac{dp}{\rho} = c\,(t) \ldots (4)$$

Equation (4) is well known famous equation called Bernoulli's equation of motion.
Once the ground work for Euler's equation and Bernoulli equation is laid down, the actual problems can be quickly solved.

Example: If a tube of small uniform bore having radii a centered at O forms 1/4th part of circle. (Denote tube by AB and B is considered below A). When end B is closed, tube is completely filled with liquid of certain density ρ. Observe that in starting $\frac{du}{dt}=\frac{2g}{\pi}$ if end B is opened immediately (here u denotes velocity) and pressure at a point P drops to $\rho g a(\sin - \frac{2\theta}{\pi})$. Solution to above problem become easy with help of Euler's equation of motion



When tube is closed pressure at ends A and B is atmospheric pressure (i.e. π)

Since $\vec{L} = \vec{g}$ and $\vec{q} = u\,\hat{s}$

By Euler's equation of motion

$$\frac{\partial(u\hat{s})}{\partial t}+u\frac{\partial(u\hat{s})}{\partial s}=-gj-\frac{1}{\rho}\left(\frac{dp}{ds}\hat{s}+\frac{dp}{dn}\hat{n}\right)$$

$$\frac{\partial(u\hat{s})}{\partial t}+u\frac{\partial(u\hat{s})}{\partial s}=g(\cos\theta\,\hat{s}+\sin\theta\,\hat{n})-\frac{1}{\rho}\left(\frac{dp}{ds}\hat{s}+\frac{dp}{dn}\hat{n}\right)$$; then

comparing coefficients of $\hat{s}$

$$\frac{\partial(u)}{\partial t}+u\frac{\partial(u)}{\partial s}=g\cos\theta-\frac{1}{\rho}\frac{dp}{ds}$$

(Clearly from figure $\sin\theta=\frac{y}{a}$ and s = a θ)

$$\frac{\partial(u)}{\partial t}+u\frac{\partial(u)}{\partial s}=\frac{g}{a}\frac{dy}{d\theta}-\frac{1}{\rho}\frac{dp}{ds}$$

$$\frac{\partial(u)}{\partial t}+u\frac{\partial(u)}{\partial s}=g\frac{dy}{d\theta}\frac{d\theta}{ds}-\frac{1}{\rho}\frac{dp}{ds}$$

$$\frac{\partial(u)}{\partial t}+u\frac{\partial(u)}{\partial s}=g\frac{dy}{ds}-\frac{1}{\rho}\frac{dp}{ds} \quad \ldots (5)$$

At point A, we have s = 0; u = 0; y = 0; p = π ; t = 0

Integrating (5) with respect to s

$$\frac{\partial(u)}{\partial t}s+\frac{u^2}{2}=gy-\frac{p}{\rho}+c(t) \ldots (6)$$

Hence $c(t)=\frac{\pi}{\rho}$

At end B, we have s= $(2\pi a)/4$; u = 0; y = a; p = π

$$\frac{\partial(u)}{\partial t}\left(\frac{\pi a}{2}\right)+0=ga-\frac{\pi}{\rho}+\frac{\pi}{\rho}$$

$$\frac{\partial(u)}{\partial t}=\frac{2g}{\pi}$$

Since $\frac{du}{dt}=\frac{\partial(u)}{\partial t}+u\frac{\partial(u)}{\partial s}$

Thus, at t=0 $\frac{du}{dt}=\frac{2g}{\pi}$

At point P

s =a θ ; u=0; t=0; y = $a\sin\theta$

By equation (6)

$$\frac{\partial(u)}{\partial t}a\theta+0=ga\sin\theta-\frac{p}{\rho}+c(0)$$

Therefore $p=\pi+\rho g a\left(\sin\theta-\frac{2\theta}{\pi}\right)$

The above example illustrate that how physical problems can be solved quickly with help of Euler's and Bernoulli's equation of motion.

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