

A Study of the Challenges and Obstacles That 11th Class Students Face when Attempting to Understand Probability Theories

Priyanka Thapliyal^{1*}, Dr. Harbans Lal²

¹ Research Scholar, Sunrise University, Alwar, Rajasthan

² Professor(Dept. of Arts), Sunrise University, Alwar, Rajasthan

Abstracts - The purpose of this study was to investigate the challenges that students in the 11th grade face when attempting to understand probability ideas. In order to accomplish this objective, ten distinct open-ended probability difficulties were presented to the 170 students in the 11th grade. Each of these issues calls for the application of a unique set of fundamental probability theories. It is qualitative study, and the descriptive method was used to analyse the information that was collected in this investigation. According to the observations, a significant number of these students struggled to find solutions to problems involving probability and were unable to successfully apply various theories of probability to these troubles. It should be noted that it was asserted that a number of students had difficulty locating all of the probabilities in these concerns without any of them being missing. In addition, it is crucial to improve reasoning skills because students usually use the technique of trial and error in conjunction with forecasting in order to solve issues involving probability.

Keywords - open-ended probability, descriptive method, trial and error method and fundamental probability theories.

-----X-----

INTRODUCTION

The need for knowledge of a variety of mathematical important subjects, such as issue, statistics, and probability, has enhanced in today's world as a direct result of technological advancements. Because of this, the concept of probability, which has been outlined among these mathematical subjects, has excited the attention of some international organisations such as NCTM (National Council of Teachers of Mathematics, 2000), and these organisations have started to emphasize the significance of people who develop their awareness of the subject matter. In addition, the content of the annual book that was released by NCTM in 2006 was constructed with probability and statistics as the foundation. The learning and instruction of probability has been the subject of a significant portion of the research carried out in the last quarter of this century. These studies have also investigated how students think about probability and how their perspectives on the topic have evolved over time (Shaughnessy, 1992). Following the occurrence of these events, a number of nations, recognising the significance of the topic, incorporated probability into their mathematics curricula (Munisamy&Doraisamy, 1998; Papaieronymou, 2010; Tsakiridou&Vavyla, 2015). The expression of the likelihood of the occurrence of events through the use of numbers is what we refer to as probability. This concept plays an

important part in both our professional and personal lives. It is a branch of mathematics that allows one to confront events from a variety of viewpoints. In everyday life as well as in the field of the humanities, one of the most common applications of probability is in judgement (Akdeniz, 2004; Kazak, 2008). When the history of probability was investigated, it was discovered that Pascal and Fermat defined probability in the year 1654. Since then, many researchers from a variety of cultures and time periods have investigated probability. Laplace was one of these individuals who played a significant part in establishing probability as the primary concept (Topdemir, 2013). In spite of the fact that probability is such an important topic, many students of all grade levels find it to be one of the most difficult topics to study (Durmus, 2004; Gurbuz, Toprak, Yapici, &Dogan, 2011; Kutluca&Baki, 2009; Tatar, Okur, & Tuna, 2008). In a similar vein, it can be challenging for a lot of students to get a firm grasp on a variety of concepts related to probability (Munisamy&Doraisamy, 1998). One of the topics that many individuals struggle with, particularly in Turkey, is probability (Sezgin-Memnun, Altun, &Yilmaz, 2010). Even though it appears to be a straightforward calculation that can be done with fractions, understanding probability takes a lot of work (NCTM, 2000). Because of this, it is extremely important to investigate probabilistic thinking, as well

as any challenges or misunderstandings associated with this topic. A wide variety of studies shed light on the challenges associated with acquiring an understanding of probability concepts.

THE SIGNIFICANCE OF THE OBSTACLE

It is anticipated that this study will give a general understanding about the opinions of students on the probability concepts that are included in the mathematics course programmes. This expectation makes this study more informative than studies that have similar aims. In addition, this study will be conducted at the eleventh-grade level, and the researcher did not come across any other studies that investigated students' difficulties and failures in solving probability problems in the literature review at this grade level in Turkey because this topic was recently added to the eleventh-grade mathematics course programmes. This study will be carried out at the eleventh-grade level. This research is important because it indicates the inadequacies in probability that eleventh-grade students have and how those students apply probability concepts. In addition, using qualitative research methods, this study will present in great detail the challenges and shortcomings that students face when attempting to understand various probability concepts. It is anticipated that these aspects will be a contribution to the field as a result of this research.

RESEARCH DESIGNS, HYPOTHESES, AND HOW THEY CORRESPOND TO ONE ANOTHER

The purpose of this study is to investigate the challenges that students face when solving problems involving probability that call for the application of a variety of different fundamental ideas of probability and to report their failures. "What kinds of problems do students in the eleventh grade have when trying to solve probability equations, and what kinds of mistakes do they make while trying to solve these equations?"

METHOD

This section contains details about the pupils who participated in the experiment, the data collected with the intention of determining the challenges and failures experienced by these students when attempting to solve problems involving probability, as well as the statistical analyses. 2.1 Participants During the first semester of the academic year 2021-2022, there were a total of 170 students who volunteered their time to participate in this research. These students enrolled in the eleventh grade at three distinct high schools in the city of Delhi. The random sampling method relies solely on the element of chance to select its samples (Yildirim&Simsek, 2005, p. 104). The participants were chosen at random in order to maintain objectivity and to ensure that each potential participant had an equal opportunity to take part in the study.

METHODS AND TOOLS FOR COLLECTING DATA

The first step was to devise a test for probability, which consisted of a total of ten open-ended difficulties. This was done by taking into consideration the 8 different acquisitions that are related to probability that are included in the mathematics course programmes. The issues were relevant to the students' lives on a daily basis and needed decision-making on the student's part. The first dilemma on the probability test was planned by drawing upon a variety of resources and text books in an effort to determine whether or not the students had a firm grasp on the ideas of sample space, experiment, and output. The 2nd problem was designed to ascertain whether or not they had an understanding of the idea of probability, and the third problem was intended to ascertain whether or not they had an understanding of the idea of complement. The topic of probability was meant to be connected to the topic of sets in the 4th problem, and the topic of equations was meant to be connected to it in the 5th problem. In a similar vein, the final three problems on the test were designed to determine whether or not the students were capable of performing calculations involving dependent and independent occurrences. After that, 9 distinct mathematics teachers looked over the problems on this test to ensure that they met certain criteria regarding level, coverage, content, and language. The difficulties of the problems were adjusted after considering the feedback from these educators. A pilot study with a total of 25 students in the grade 11 was conducted before various teachers checked the final revision of this probability test. The responses of the students were organised into categories for the purpose of this pilot study, and the data that was collected was compiled and organised according to the categories that were determined. As a result, a preliminary draught of the evaluation form that will be utilised in the evaluations was prepared. The problems on the test were examined in this manner in order to determine how easily they could be solved as well as whether or not they were even possible.

DATA COLLECTION AND ANALYSIS

The research questions pertaining to probability were discussed in written questionnaires that were distributed to the students and given to them to answer while being supervised by their respective mathematics teachers. The students needed approximately half an hour to work through all of the problems in the probability test. It was presumed that these students in the eleventh grade incorporated their actual expertise and abilities into their answers to the issues. The descriptive analysis method was utilised in order to evaluate the students' responses to the probability problems. The characteristics of descriptive analysis include having a methodical and precise description of the information trying to explain and analysing these descriptions,

investigating the cause-and-effect connections, and coming to some tangible conclusions. In most cases, the results that are obtained are associated with and analyzed in terms of themes, and then predictions are made (Yildirim&Simsek, 2005, p. 224). In the context of this investigation, the information that was gleaned from the students' responses to the probability problems was portrayed in a descriptive manner. This was accomplished by adhering to the data in its original form to the greatest extent possible, and, when it was deemed appropriate to do so, by directly quoting what the participating students had to say. Additionally, interpretations of the research results were included (Walcott, 1994). Within the confines of this field of view, an analysis of the obtained data was carried out by first developing a framework, then handling the data in accordance with the theoretical model, and finally explaining and analyzing the results. Determine the students' difficulties and missteps in trying to solve the probability troubles during the phase of establishing a framework. This will create the most basic outline for the data analysis. During the stage devoted to the subject to regulation, the data were classified based on the various themes that had been determined and presented in accordance with this structure. At this point in the process, the study can be broken down into three distinct categories: no answer, incorrect answer, and correct answer. During the stage of the data's description and interpretation, the obtained data were read and arranged according to the themes that were decided upon. After that, the data that had been arranged were described, and the conclusions were interpreted in light of these themes. In this part of the process, the researcher read and evaluated the data without any outside input. When the study data could not be categorised according to the concepts decided in the pilot study, discussions with a mathematics instructor were held to determine which answer should be categorised according to which theme. After that, the reorganisations were carried out in accordance with the findings that were discovered. After that, the themes that had been arranged were explained, and the conclusions that had been attained were interpreted in light of these concepts. After that, these information were put through a series of specific methods in order to be reduced to numbers, including the calculation of frequency and percentage values. In conclusion, the results of the study were backed by direct quotations whenever it was necessary to do so.

RESULTS

In this section, the comprehensive research results that were reached as a consequence of the statistical tests that were made with the goal of investigating the challenges and inadequacies that the participant 11th students had in trying to solve probability problems are included, as well as the various interpretations that have been made for these study results. First, a total of 170 students' responses to the probability difficulties were classified, and the % and frequency values for this categorization are shown in Table 1 below.

Table 1: Percentage and frequency values related to the answers given to the problems

Ques.	Correct answer		Incorrect answer		No answer	
	f	%	f	%	f	%
1	21	14.8	111	78.2	10	7.0
2	79	55.6	54	38.1	9	6.3
3	8	5.6	104	73.3	30	21.1
4	43	30.2	82	57.8	17	12.0
5	50	35.2	57	40.1	35	24.7
Total	201	14.14	408	28.75	101	7.11

In this section of the study, the researchers analysed the responses provided by the students to each study topic. The answers that the students provided to the 1st problem were analysed and is included in the research as follows: "Find the sample space, experiment, and output in the problem of 'Ayse will choose one skirt and one jacket from among the two red skirts, three blue skirts, three red jackets, and one blue jacket that are in her wardrobe.' The purpose of this step was to determine whether or not the students had a firm grasp on the ideas of sample space, experiment, and output. Find out what the odds are of picking a jacket and skirt that are the same colour. According to the findings of the evaluations that were carried out, a total of 111 of the 11th students who participated gave an incorrect response to this question. The percentages and frequency values of these student responses were computed for you.

Table 2: Analysis of results related to the incorrect answers given to the first problem

First problem	f	%
Being unable to understand/incorrectly understanding the problem	71	50.0
Incorrectly determining the sample space	35	24.7
Being unable to consider all the conditions	5	3.5
Total	111	78.2

"When a pair of dice are tossed up, what is the probability that the sum of the numbers coming on top surface is less than 10?" is the second problem in the study, and its purpose was to determine whether or not the concept of probability was comprehended. It was stated as "What is the probability that the sum of the numbers coming on top surface is less than 10?" When the solutions that 170 students had provided to this problem were analysed, it was discovered that rhythmic counting was the method that was utilised by 40.1% of the respondents in order to overcome the problem. On the other hand, some of them (14.1%) found the set that complemented each other, and a few of the students (1.4% of the total) determined the probability. However, as a result of the analyses

done and presented in Table 3, it was determined that a total of 54 of the participating students had provided incorrect answers to this problem, as well as the percentage and frequency values were computed for these responses.

Table 3: Analysis of results related to the incorrect answers given to the second problem

<i>Second problem</i>	<i>f</i>	<i>%</i>
Doing rhythmic counting incompletely	19	13.4
Being unable to understand/misunderstanding the problem	17	12.0
Being unable to consider all the conditions	8	5.7
Calculating the probability greater than 1	6	4.2
Making calculation errors	2	1.4
Writing the sample space incorrectly	1	0.7
Being unable to find/incorrectly finding the complement of a set	1	0.7
Total	54	38.1

The third research problem in the study was designed with the intention of determining whether or not the idea of complement was understood, and it was stated as follows: "Four economists, three chemists, and two engineers will sit around a round table." Determine the likelihood of people from the same line of work being seated next to one another. The responses that the participating students had provided to this problem were analysed, and it was discovered that those students who had provided an answer that was correct had done so by performing calculations involving probability or by determining the complement of a set.

Table 4 contains the responses given by the students who took part in this investigation. Based on these responses, it was determined that a total of 104 of the participating students got this problem wrong, as well as the percentage and frequency values that were calculated for these answers.

<i>Third problem</i>	<i>f</i>	<i>%</i>
Being unable to consider all the conditions	31	21.9
Being unable to understand/misunderstanding the problem	31	21.9
Calculating the probability greater than 1	29	20.4
Being unable to find/incorrectly finding the complement of a set	9	6.3
Making calculation errors	4	2.8
Total	104	73.3

The students in the eleventh grade were given the fourth research problem, which associated the topic of probability with the topic of sets as follows: "Of the 25 students in a class, 15 students prefer mathematics, 12 students prefer other subject, and 6 students prefer both mathematics and other subject." What are the odds that a student chosen at random from the classroom is one who does not prefer mathematics or any other subject? After looking over the responses, it was discovered that a few of the students (30.3%) had correctly answered the question. Some of the students (4.9% of them) solved the problem by calculating the complement of a set, and some of the students (1.4%

of them) solved the problem by doing the probability calculation directly. However, the vast majority of the students (23.9% of them) solved the problem by using a Venn diagram. On the other hand, it was discovered by looking at Table 5 that a total of 82 students answered this problem in an incorrect manner. The percentages and the frequency values of the responses from the students were calculated.

Table 5: Analysis of results related to the incorrect answers given to the fourth problem

<i>Fourth problem</i>	<i>f</i>	<i>%</i>
Being unable to understand/misunderstanding the problem	60	42.3
Calculating the probability greater than 1	11	7.8
Making calculation errors	7	4.9
Finding the intersection set incorrectly	2	1.4
Being unable to calculate all the probabilities	2	1.4
Total	82	57.8

The 5th problem in the research was designed to see how the students connected the topic of probability with eqs. It was described as follows: "There are three times more red balls than there are black balls in bag A, and there are twice as many red balls as there are black balls in bag B." The purpose of this problem was to evaluate how the students connected the particular topic of probability with equations. When one ball has been drawn from each bag, what is the likelihood that the balls drawn will be of varying colours? After examining the solutions that the students had provided to this problem, it was discovered that the majority of the participants had arrived at the correct response by using probability calculations. On the other hand, it was comprehended from Table 6 that a total of 57 students responded this problem incorrectly, as well as the percentages and frequency values that were determined for these responses.

Table 6: Analysis of results related to the incorrect answers given to the fifth problem

<i>Fifth problem</i>	<i>f</i>	<i>%</i>
Making calculation errors	19	13.4
Being unable to understand/misunderstanding the problem	18	12.7
Being unable to calculate all the probabilities	11	7.5
Calculating the probability greater than 1	7	4.9
Being unable to consider all the conditions	2	1.4
Total	57	40.1

DISCUSSIONS

The findings demonstrated that the students' failures in the estimations for some probability difficulties included in the research led us to the conclusion that they struggled with fractions, calculations involving percentages, and the four arithmetic operations that are performed with numbers. This finding is consistent with the findings obtained in the research conducted by Jones et al. (1996) and Sezgin-Memnun, Altun, and Yilmaz (2010), which found that

the students had difficulty, in particular when it came to comprehending and applying the idea of sample space.

Some of the attendees lacked an understanding of the boundaries of probability, and others struggled to grasp basic concepts of probability, such as the experiment, the output, the separate event, the dependent and independent events, and the complementary event. Students have a propensity to have many misunderstandings in regards to probability, as well as in other areas of mathematics. This suggests that students have a tough time grasping the fundamental concepts of probability. This finding lends credence to the findings of previous research (Batanero & Serrano, 1999; Gurbuz, 2006; Sezgin-Memnun, Altun, & Yilmaz, 2010), which suggest that students have a difficult time perceiving and comprehending probability concepts. At the same time, it lends credence to the findings that were obtained from the research carried out by Sezgin-Memnun, Altun, and Yilmaz (2010), as well as Tunc (2006), which indicated that students struggled to comprehend the fundamental ideas underlying probability.

On the basis of the observations made and the findings that were obtained, the following recommendations were made:

- When trying to study probability in mathematics classes, there should be more of an emphasis placed on the studying of the fundamental concepts of probability.
- It is essential to communicate the data achieved from sample space to students in a meaningful way by providing instances, which students can relate with everyday life. It is recommended that students not be required to memorise information when they are learning about probability.
- During the method of teaching probability in mathematics classes, particular focus should be placed on the utilisation of concrete materials like coins, dice, and various geometrical shapes.
- The method of learning probability can begin once students in middle school and high school have strengthened their preparedness levels and recollected the necessary prior knowledge for resolving problems involving probability. It is possible to conduct an investigation into the role that this subject plays in the students' learning of probability.

REFERENCES

1. Gurbuz, R., Birgin, O., & Catlioglu, H. (2012). Comparing the probability-related misconceptions of pupils at different education levels. *Croatian Journal of Education*, 14, 307-357.
2. Gurbuz, R., Toprak, Z., Yapici, H., & Dogan, S. (2011). Ortaogretim matematik mufre

datindazorolarakalg ilanank onul arvebunlarinnedenleri. Gaziantep Universitesi Sosya l Bili mler Dergisi, 10(4), 1311-1323.

3. Kazak, S. (2008). Ogrencilerin olas ilik konularinda kikkav ramya nilgilariv eogre nmez orluklari. In M. F. Ozmantar, E. Bingolbali, & H. Akkoc (Eds.), *Matematiksel Kavram Yanilgilarive Cozum Onerileri* (pp. 121-150). Ankara: Pegem Publications.
4. Kutluca, T., & Baki, A. (2009). 10. Sinif matematika tikdersinde zorlanila nkonular hakkinda ogrencilerin, ogretmenad aylarinin veogre tmen leringorusl erinininc elenmesi. *Kastamonu University Education Journal*, 17(2), 609-624.
5. MEB (Ministry of Education). (2011). Ortaogretim Matematik (9, 10, 11 ve 12. Siniflar) Ders iOgretim Programi. Ankara: MEB.
6. Topdemir, H. G. (2013). Bilimse ldevrimy uzyu llarindamatematik. *BilimveTeknik*, 552, 88-89.
7. Tsakiridou, H., & Vavyla, E. (2015). Probability co ncepts in primary school. *American Journal of Educational Research*, 3(4), 535-540. <https://doi.org/10.12691/education-3-4-21>.
8. Yildirim, A., & Simsek, H. (2005). *Sosyalbilimle rdenite larasti rmayontemleri*. Ankara: Seckin Publications.

Corresponding Author

Priyanka Thapliyal*

Research Scholar, Sunrise University, Alwar, Rajasthan