Fermionic Spectrum With Spontaneous Symmetry Breaking

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Abstract - These gravity duals are amenable to include fermionic fields, allowing for the investigation of fermionic excitations in the dual system. Research on Fermi surfaces using holography was originally published in. We've looked at the spectral function while considering fermions that emerge from supergravity theory. It turns out that the system can tolerate a gapped spectrum. As can be seen from the dual field theory analysis, the gapped spectrum is caused by fermionic operators involving bosons with a non-zero expectation value. We have evaluated fermions of a single sector against this domain wall and investigated the dispersion relation of their modes. In the positive zone, the dispersion relation is hyperbolic for smaller fermion charges, leading to a gapped spectrum.

Keywords - Fermionic, Spectrum, Spontaneous, Symmetry and Breaking

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INTRODUCTION

Gauge/gravity duality and related holographic notions have been central to the study of basic physics in the past twenty years. The essential notion is that a theory defined on a (usually) higher-dimensional space or spacetime ('the bulk') might be equivalent to a theory defined on a lower-dimensional space or spacetime ('the border') by a factor of (D 1). A correspondence (or "dictionary") between the concepts of one theory and those of another expresses this equivalence. Some of the reasons for this are that (i) the concepts that the dictionary declares to 'translate' into each other can be strikingly different; for example, a direction in the bulk spacetime is translated as the direction of the renormalization group flow in the boundary theory; and (ii) problems in the strong coupling regime of one theory can be solved more easily in the weak coupling regime of the other theory.

Holographic It was in five-dimensional maximally gauged supergravity, whose four-dimensional dual field theory is N = 4 Super-Yang-Mills (SYM), that the top-down Fermi surface was first found. In this supergravity theory, against the backdrop of a black hole solution, only spin-1/2 fermions that do not pair to gravitini were studied. At the black hole chemical potential's limiting value, they discover that the dual fermionic operators are in the non-Fermi domain but approach the marginal Fermi liquid phase. Excitation width of quasi-particles was also investigated. These investigations were then broadened to include more maximum gauged supergravities. In specifically, it was used to the four-dimensional version of supergravity, the dual of the three-dimensional Aharony-Bergman-Jafferis-Maldacena (ABJM) theory. These were low-temperature studies, and later, analysis of the dual field theory's scalar and spinor operators appeared in.

Similar discussions of Fermi surfaces were presented in. The gravity duals are black holes in the aforementioned works. Nonetheless, the majority of the solutions discussed in that article have zero-point entropy 1. For T 0, the entropy does not go to zero. However, there is an intriguing group of gravity-side backgrounds that don't have this issue. They produce spontaneous violation of U (1) gauge symmetry in the dual theory, which is analogous to condensation of charged scalar. These characteristics, known as domain walls, are regarded the zero-temperature limit of holographic superconductors. For a zero-temperature holographic superconductor in a condensed phase, a study of fermions has been published that displays a spectrum comparable to that observed in the APRES experiment. Several publications arose that attempted to explain the process that caused the gapped spectrum. In particular, the gapped spectrum is likewise produced by self-coupled Majorana fermions interacting with a scalar.

LITERATURE REVIEW

Gautam, V. et.al (2022), The theory of gravity the emergent geometry of gravity/gravity duality relies heavily on matrix degrees of freedom. Here, we show how the entanglement between matrices of degrees of freedom may be analogized to gravitational entanglement. Our approach, which we term "matrix entanglement," is distinct from the "target-space entanglement" that has been created and studied by other groups in recent years. Several classes of quantum states are discussed where our approach may be useful. In a regularized, nonperturbative context, a fuzzy sphere may be used to explain the usual spatial entanglement in two-brane or five-brane world-volume theory. For a microscopic black hole in AdS_5S5 that may evaporate uncoupled to a heat bath, our technique also offers a gravity theory origin of the Page curve. The degrees of freedom that are still restricted play important roles in partly liberated governments.

Liza Huijse et.al (2018) The zero-temperature phases of compressible quantum matter are surveyed in detail. These are phases in which the projected value of a globally conserved U (1) density, Q, fluctuates smoothly as a function of parameters. Phases with preserved global U (1) and translational symmetries are predicted to have Fermi surfaces thanks to the connection between the volumes encompassed by these surfaces and hQi established by the Luttinger theorem. We briefly review the ideas of the interactions between bosons, fermions, and the gravitational field that produce these states. Landau's Fermi liquid theory predicts that the Fermi surfaces of certain phases will exhibit singularities, whereas the Fermi surfaces of other phases would not. It is argued that the two paradigmatic supersymmetric gravity theories underlying gravity-gravity duality, the ABJM model in d = 2 and the N = 4 SYM theory in d = 3, exhibit compressible phases similar to those found in models applicable to condensed matter systems when chemical potentials (and other deformations allowed by the residual symmetry at non-zero chemical potential) are applied to them.

Nishal Rai (2019) Strongly coupled d-dimensional field theories may be related to weakly coupled d + 1 dimensional gravity theories, and vice versa, as suggested by the Gravity /Gravity duality. In many fields, including gravity theory, hydrodynamics, condensed matter theories, etc., this holographic duality offers very potent tools for disentangling and investigating tightly related regimes. In this thesis, the Gravity / Gravity duality is used to explain some behaviors in tightly connected condensed matter systems. When perturbative approaches of quantum field theory are not applicable, the dynamics of such systems may be captured using formalisms within the setting of condensed matter theory, such as Fermi liquid theory. However, in the 1980s, it was discovered that certain materials exhibited characteristics at odds with those anticipated by the Fermi liquid hypothesis, and these substances were subsequently labeled as non-Fermi liquids. Materials approaching a quantum phase transition include heavy fermions and high (Tc) cuprate superconductors. The work's goal is to investigate the functioning of such systems by using the Gravity /Gravity duality. Starting with a well understood dual field theory and a string-theory inspired gravity theory, we have utilized a top-down method.

Dumitrescu, Thomas et.al (2015). The gauge/gravity duality establishes new bridges between quantum mechanics and general relativity. As a result, many different branches of theoretical physics now have access to novel ideas and methods for problem-solving. This is the first comprehensive textbook on this issue, and it will be useful to graduate students and researchers in string theory, particle physics, nuclear physics, and condensed matter physics. This textbook provides a comprehensive introduction to the key ideas of gauge/gravity duality, focusing on both the basic elements and the applications. Specifically, it clarifies how the conjectured map from string theory to the AdS/CFT correspondence comes to be. Applications to other fields are then described, including those dealing with finite temperature and density, hydrodynamics, QCD-like theories, the quark-gluon plasma, and condensed matter systems.

DOMAIN WALL SOLUTION

covers the discussion of the shortened version of the bosonic action of maximum gravity d supergravity. For this truncated theory, we may find a distinct class of solutions, called domain wall solutions, in addition to the black hole solutions. Since just two Cartan gravity fields have been taken into account, the Lagrangian for this theory is provided. $A^{12}\ \mu = A^{(1}\ \mu^{)}$ and $A^{34}\ \mu = A^{(2}\ \mu^{)}$. Considering a diagonal scalar vielbein V^{i} will lead to $U\ (1)^2$ The situation presented in the result of the symmetry of gravity. To achieve our goal of having one of the U(1) broken in this model, we explore the following ansatz for the scalar vielbein background.

$$V_I^i = \exp[\phi_2 Y_2]\exp[\phi_1 Y_1 + \phi_3 Y_3], \qquad Y_1 = \mathrm{diag}(1,-1,0,0,0),$$
$$Y_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \oplus \mathrm{diag}(0,0,0), \qquad Y_3 = \mathrm{diag}(0,0,1,1,-2),$$

where Y_1, Y_2 and Y_3 comprise the group SL (5)'s generators. It turns out that the bosonic action is optimal for such a scenario.

$$2\kappa^2 e^{-1}\mathcal{L} = R - \frac{m^2}{2}V(\phi_1,\phi_3) - 2(\partial\phi_1)^2 - 2\sinh^2 2\phi_1(\partial_\mu\phi_2 + gA_\mu^{(1)})^2 - 6(\partial_\mu\phi_3)^2$$
$$-(F_{\mu\nu}^{(1)})^2 - e^{4\phi_3}(F_{\mu\nu}^{(2)})^2,$$
$$\text{where} \quad V(\phi_1,\phi_3) = -(e^{2\phi_1} + e^{-2\phi_1} + 2e^{-2\phi_1} + e^{4\phi_3})^2 + 2(e^{4\phi_1} + e^{-4\phi_1} + 2e^{-4\phi_3} + e^{8\phi_3}).$$

The given Lagrangian may be made non-symmetric by setting 2 = 0 μ to violate the U (1) symmetry associated with $A^{(1)}$. From (5.2.1) As we can see, this is analogous to picking unitary gravity for the closet. We may further reduce the number of equations involved by setting them to fewer values from the Lagrangian resulting equations (5.2.2). $\varphi_3 = 0$ and $A^{(2)} = 0$. Equations of motion derived from the aforementioned Lagrangian are compatible with this description. The possible $V(\varphi_1, \varphi_3)$ will reduce to

$V(\varphi_1) = (2\cosh 2\varphi_1 - 3)^2 - 16.$ (5.2.3)

The extrema of this potential (5.2.3) lie at $\varphi_1 = 0$ $\phi_1 = \frac{1}{2}Log(\frac{3\pm\sqrt{5}}{2})$. We will think about solutions for the domain wall if the scalar 1 is somewhere in the middle of these two extremes. The metric and gravity field ansatzes are selected as follows to achieve such an interpolating solution.

$$ds^2 = e^{2A}(-hdt^2 + d\vec{x}_5^2) + \frac{dr^2}{h}, \qquad A^{(1)} = B_1 dt.$$

We have evaluated the conductivity using the equation provided in after numerically solving according to the boundary condition The relationship between conductivity and frequency is seen in Fig. 1 To investigate the conductivity's low-frequency behavior, ω, following we will introduce F,

$$\mathcal{F} = -he^{4A}\frac{a_x^*\partial_r a_x - a_x\partial_r a_x^*}{2i},$$

where ∂_rF = 0, ω thus, given It follows from that in the IR limit, F is not a function of. Conductivity's "real" component may be calculated as

$$Re(\sigma) = \frac{\mathcal{F}}{4h_{UV}}\frac{1}{\omega|a_x^{(0)}|^2},$$

The reliance on may be calculated by first establishing the dependence of a(0) on. With respect to the area ω $L_{IR}Log(\omega L_{IR}) << r << rIR$ One may demonstrate that the geomtry deviates just little from Ad_{SIR} in cases,

$$a_x \sim -i\frac{\Gamma(\triangle_B + 2)}{\pi}\left(\frac{2}{\omega L_{IR}}\right)^{\triangle_B+2} Z_x(r),$$

where at the IR region, $\lim_{r\to-\infty} e^{-\triangle_B r/L_{IR}} Z_x(r) \to 1.$ Since the 2 component in eq. becomes small for big r, it follows that the ω2 dependency of a does not change much even at huge r. $a^{(0)} = \lim\ a(r) \sim \omega_\mu^{-(\Delta B+2)}$ I t follows

$$Re(\sigma) \sim \omega^{2\Delta_B+3}.$$

For small, as can be shown in Figure 1, the ω reliance of Re(σ) coincides with this. In the ultraviolet AdS7 shape, the real component of the conductivity ω behaves as 3 for big.

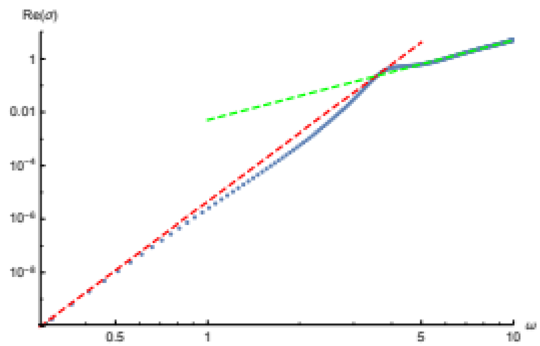

Figure 1: Re(σ) vs. ω

FERMIONIC ACTION

The gravitino A and spin-1/2 field that make up the fermionic content of N = 4 gravity d supergravity in seven dimensions have been studied at length in earlier λ^A. In this domain wall background, we will be only considering those fermions which do not couple with gravitino. The Lagrangian consisting of only spin-1/2 fields λ^i which we see in (3.3.3). The covariant derivatives are different from those in since our starting point is different. In this context, the covariant derivatives are defined as

$$D_\mu\lambda^i = \nabla_\mu\lambda^i - ig\cosh 2\phi_1[A_\mu^{(1)}(J^{12})^i_j)\lambda^j + (A_\mu^{(1)}S^{12})\lambda^i],$$

where J^{12} and S^{12} are the vector and spinor representations' respective U (1) gravity group generators. The spin connection is included in the covariant derivative as shown below.

$$\nabla_\mu = \partial_\mu - \frac{1}{4}(\omega_\mu)_{ab}\Gamma^{ab}.$$

Coupling of gravitinos is reflected in the Lagrangian terms. Ψ_μ andspin-1/2 fields λ^i are given in (3.3.6).

detail the simplified notation for SO (5) vector indices that we will be using. This SO (5) notation allows for the orderly arrangement of's 16 constituent parts as $\lambda^{1\pm}$ (s_{12}, s_{34}) and $\lambda^{2\pm}$ (s_{12}, s_{34}).

We are especially curious in fermions that do not interact with gravitino. The supplementary U (1) (inside the $A^{(2)}$) remains unbroken and gravitino has charges ±1 with respect to it. Therefore, spinor components with total charge ±3 2 with respect to

second U (1) do not couple to gravitino. So λ^{2-}(s_{12}, $\frac{1}{2}$) and λ^{2+}(s_{12}, $-\frac{1}{2}$), with s_{12} =

±$\frac{1}{2}$ we focus on the special circumstance when these four fermions are unbound from the gravitino. For the current value of V i, an explicit calculation demonstrates that all the other fermions in 16 couple to the gravitino.

Dirac equation for these fermions λ_i can be written as

$$(\Gamma^\mu D_\mu + \frac{m}{4}(5 - 2\cosh 2\phi_1) + i\frac{s_{12}}{2}\Gamma^{\mu\nu}F^{(1)}_{\mu\nu})\lambda = 0,$$

$$\text{where,}\quad D_\mu\lambda = \partial_\mu\lambda - ig(q\cosh 2\phi_1 A^{(1)}_\mu)\lambda,\quad F^{(1)}_{\mu\nu} = \partial_\mu A^{(1)}_\nu - \partial_\nu A^{(1)}_\mu,$$

the charges q s12 gives us information on the gravity field, but we've ignored it in favor of a more general description in our research. This scalar field φ_1 determines both the mass and charge term. At IR limit the mass is given by $m/2$ where the scalar $\varphi_1 = \varphi_{IR}$, while at the UV limit $\varphi_1 = 0$ and the mass turns out to be $3m/4$.

Γ-matrices are chosen as follows

$$\Gamma^{\hat{t}} = \begin{pmatrix} \Gamma_1^{\hat{t}} & 0 \\ 0 & \Gamma_1^{\hat{t}} \end{pmatrix},\quad \Gamma^{\hat{r}} = \begin{pmatrix} \Gamma_1^{\hat{r}} & 0 \\ 0 & \Gamma_1^{\hat{r}} \end{pmatrix},\quad \Gamma^{\hat{i}} = \begin{pmatrix} \Gamma_1^{\hat{i}} & 0 \\ 0 & \Gamma_1^{\hat{i}} \end{pmatrix}$$

$$\Gamma_1^{\hat{t}} = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix},\quad \Gamma_1^{\hat{r}} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},\quad \Gamma_1^{\hat{i}} = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix},$$

σ1, σ2 and σ3 and are the identity matrix, the Pauli spin matrices, respectively. The relevant -matrices all share the same block diagonal, so we can choose any one of them., λ =(Ψ, χ)^T where both the 4-component spinors Ψ and χ satisfies the same equation. In this regard, we will consider only the upper component Ψ. Redefining the spinors with the right prefactor and picking the right spinor will break the spin link. Ψ = (Ψ^+, Ψ^-)^T, with each part meeting the conditions of the equation:

$$(\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\Psi^+ + ie^{-A}[k\sigma_1 - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} - s_{12}B_1')i\sigma_2]\Psi^- = 0,$$

$$(-\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\Psi^- + ie^{-A}[k\sigma_1 - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} + s_{12}B_1')i\sigma_2]\Psi^+ = 0.$$

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hese two component spinors are further written as $\Psi^\pm$ = ($\Psi_1^\pm$, $\Psi_2^\pm$) Then get the equations for all the parts,

$$(\sqrt{h}\partial_r - \frac{m}{4}(5 - 2\cosh 2\phi_1))\Psi_1^- - ie^{-A}[k - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} + s_{12}B_1')]\Psi_2^+ = 0,$$

$$(\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\Psi_2^+ + ie^{-A}[k + (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} - s_{12}B_1')]\Psi_1^- = 0.$$

It can be shown that the Dirac equation may be decomposed into two linked equations for the two parts. (Ψ_1^-, Ψ_2^+). The equations for the remaining two parts are the same.

$$(\sqrt{h}\partial_r - \frac{m}{4}(5 - 2\cosh 2\phi_1))\Psi_2^- - ie^{-A}[k + (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} + s_{12}B_1')]\Psi_1^+ = 0,$$

$$(\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\Psi_1^+ + ie^{-A}[k - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} - s_{12}B_1')]\Psi_2^- = 0.$$

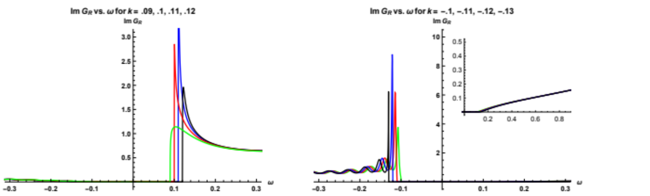
2 2

The only difference between these two sets of equations is the sign of ω, q and s_{12}. Therefore, we need only investigate one set of equations, and that is the first set We were unable to discover an analytic solution for these equations, therefore we will be employing a numerical approach with suitable boundary conditions to solve them. However, one may get analytic formulas for the solutions' behavior in the IR and UV limits.

NUMERICAL RESULT

We have discussed fermions so far. $\lambda^{2\pm}$ ($\frac{1}{2}$, $\pm\frac{1}{2}$) and displayed the boundary dual CFT spectral function for its operator. First-U (1) fees are calculated as q = ±$\frac{1}{2}$ and the coefficient of the Pauli term is ±$\frac{1}{2}$. In Fig. 2 we have plots of spectral function $Im(G_R)$ vs. ω given a range of k, where the two charges are distinct q = 1/2 and −1/2.

For positive q, which can be observed in Fig. 2 a, all peaks lie on the positive side of ω. Height of the peak is highest for k = 0.11, ask moves away from it, the height of The climax is lowering. On the - side of, it forms a bump ω. Peaks emerge on the minus side of for negative q, as seen in Fig. 2b, with the greatest peak ω. height at k = 0.12. The height of the apex decreases as k deviates from this value of k. Once again humps appear, this time on thepositive side of ω. Since the bumps in the main image are obscured, we have included an expanded version of it ω in the inset for values of positive. Both situations exhibit a depletion in the spectral function near ω = 0, suggesting a spectral gap. We have searched the spectral function for more large q values. For this larger charge, we observe that the depleted zone is still present. As the charge q rises, however, it seems that more peaks form and their heights grow.



(a) ImG_R vs. ω for $q = 1/2$ (b) ImG_R vs. ω for $q = -1/2$

Figure 2: Spectral function for fermionic mode with: Left: $k = .09$ (green), $k = .1$ (red), $k = .11$ (blue), $k = .12$ (black). Right: $k = -.1$ (green), $k = -.11$ (red), $k = -.12$ (blue), $k = .13$ (black).

The fermionic operators' spectrum may be gapped if there isn't enough charge or if there's a Pauli term. Asymptotically, the charges of fermions in the U (1) gravity group are given by $q = \pm\frac{1}{2}$ .To test if this gap in the spectrum is charge-dependent, we have artificially adjusted q to q = 2 and disabled the Pauli term by setting it to zero. This chasm, seen graphically in Fig. 3 When $|q|$ greater than q = 1/2, the ω peaks emerge on the positive side of and are more prominent. Unfortunately, at this q value, humps and little peaks still persist.

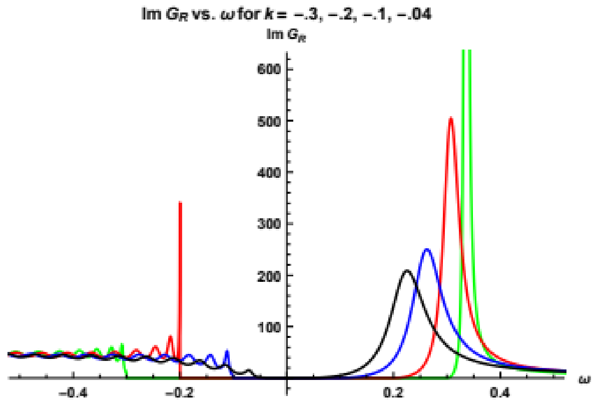


Figure 3: ImG_R vs. ω for $q = 2$

CONCLUSION

Naturally, novel methods are required for such systems. Since the issues of highly coupled field theories may be translated into those of weakly coupled gravity theories, gauge/gravity duality has been shown to be extremely effective. We have addressed fermions that decouple from gravitino of this supergravity theory using this domain wall as a backdrop to explore fermionic behavior in such zero-temperature limit of a superconductor. The spectral function of the related dual operators has been analyzed by us. The superconductor should have a gapped spectrum in the backdrop.

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