

A Study of the Fuzzy approach to Multiple Attribute Decision Making used in Investment Company

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Abstract - The typical approach to grey-area analysis is used in this method. The fuzzy eigenvector of the fuzzy attribute evaluation space is used in the proposed enhanced fuzzy AHP technique, which significantly outperforms previous similar algorithms. It has a great resolution and is completely impartial. This technique takes into account the fact that any individual or group making a decision may have limited information about the input parameters of a choice that must satisfy several criteria. This article's strategy is easy to understand and use, and it demands less dogmatic thought from decision-makers than competing strategies. Weight inputs may be a fuzzy number between two intervals. Fuzzy values representing generic intervals may be used to submit alternative performance evaluations. After the evaluations for both convenience and effectiveness have been normalized, a composite utility value may be calculated within a certain range.

Keywords - Fuzzy Approach, Multiple Attribute, Decision Making, Investment Company

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1. INTRODUCTION

When investing in a volatile market, it's important to put more weight on strategies that boost returns while reducing exposure. When Harry Markowitz created the variance portfolio model in 1952, he became generally recognized as the father of contemporary portfolio theory. This idea is a useful framework for keeping track of monetary obligations. Markowitz's Portfolio theory was heavily modified by both the Capital Asset Pricing model proposed by Sharpe and the Arbitrage Pricing theory proposed by Ross before they were applied to the problem of making investment choices. Real data are used to illustrate the impact that various strategies have on an investor's final choice.[1]

However, there are still many unknowns that must be taken into account while making financial investments. The Multi-Attribute Decision Making (MADM) paradigm may be used to investing choices. Researchers from all across the globe have examined MADM. When it comes to MADM, there are two basic problems that require fixing. First, one must learn how to accurately describe the enigmatic components, and second, one must learn how to successfully combine those components.[2]

The fuzzy set approach, the rough set technique, the probability method, and interval numbers are all possible ways to highlight the aspects that contribute to uncertainty. Interval numbers are a useful tool for

dealing with uncertainty since their value range is broader than that of a real number and they have straightforward representations. As a result, the interval values pointed to different kinds of uncertainty that needed to be accounted for in budgeting. To solve the second issue, the Dempster-Shafer (D-S) theory proved useful.[3]

1.1 Types and sources of fuzziness

Types of Fuzziness

One of the most basic difficulties in information processing is the presence of imprecision or fuzziness in the provided data. Although it is ideal to have precise and trustworthy information, there are times when less trustworthy sources must do. It is possible to broadly categorize data based on how precise or vague it is:[4]

Intrinsic fuzziness

One of the hallmarks of human knowledge is our capacity to process and communicate complex information using natural language. However, the perception of certain information or a situation may vary depending on the environment and preference inherent to human subjectivity.

Informational fuzziness

This haze arises from the gap between the sheer volume of data available and the limited ability of the human mind to process and synthesize all of that data into a coherent set of evaluation criteria. In a complex decision situation, it is unlikely that a single criterion would be sufficient for making a determination; rather, a large number of criteria describing the intended purposes will likely be used. Because there is so much information available to use as criterion, the complexity of the issue is certain to increase. There comes a point when an increase in complexity makes it impossible for a human to make a judgment that is both precise and meaningful.

Relational fuzziness

As a result of the frequent involvement of non-dichotomy qualities in the construction of implicit expressions, some phenomena or connections might be hazy, leading to relational fuzziness. Phrases like "not so much greater than average value," "about the same," "above limit level," and "if the net present value continues favorably high, then the project is quite viable" all exhibit relational fuzziness and are used often in ordinary conversation. Decision analysis relies significantly on models that account for the inherent uncertainty in human judgment, and fuzzy set theory provides an ideal framework for doing so.

Sources of Fuzziness

Imprecision and fuzziness can arise from a number of different sources, some of which are listed below.

Lack of information

Insufficient data is likely the primary cause of inaccurate estimates. There will be a quantitative shortage of knowledge if the decision maker does not know which of the potential states of nature will occur. One possible source of the uncertainty is the difficulty in accurately measuring the relevant variables. The matter is made more difficult by the fact that the word "measuring" is used in a variety of ways depending on the context. It's possible to get clean data sometimes, but it's either too expensive or takes too much time. Yet, there are situations where the decision maker utilizes an estimate of the precise data or even incorporates linguistic descriptors because they are not interested in acquiring the exact number, such as when the data is highly sensitive.

Abundance of information

A statistical report on a country's economy could contain information like the country's production, import and export, inflation rate, interest rate, per capita income, labor force, unemployment rate, and consumer price index. The economy's complicated hierarchical structure makes it difficult to provide a straightforward response to the issue of whether or not the climate is conducive to investment. There is a lot of information to work with, yet there is still some mystery

because of the limitations of human cognition. To deal with this kind of situation and make decisions more easily, the necessary data are usually turned into intelligible information in a single criteria.[5]

Conflicting evidence

Inconsistent evidence about a situation might also contribute to the air of mystery around it. An increase in information may not lessen ambiguity but may increase conflict and complexity if the two types of current information are at odds with one another. Clearly, there is a wide range of potential causes for this discrepancy in evidence. It is preferable to double-check the accuracy of the existing information rather than collecting new data in order to lessen doubt caused by contradictory facts. But, in other circumstances, the disagreement can be reduced and progress toward more confidence can be made by omitting some bits of information.

1.2 Goals of fuzzy set theory

The primary goals of fuzzy set theory are as follows:

Modelling of "Uncertainty": Fuzzy set theory has undoubtedly been used to model "uncertainty" in language. Yet, "uncertainty" that is based on chance may be represented by probability theory.

Relaxation: Most traditional approaches to optimization and modeling have a binary structure, in which success or failure, optimality or suboptimality, and identity or non-identity are treated as distinct categories.

Complexity reduction: When there is too much data, it is difficult for humans to take it all in at once and make sense of it. Hence, fuzzy set theory employs linguistic variables to simplify data to a level where it can be understood by humans and where the most crucial structures can be readily identified.

Meaning preserving reasoning: Knowledge-based systems, such as expert systems, frequently consult humans for advice before running their findings via inference engines to find workable answers. The information is often organized into if-then rules. The computer merely interprets the numerical values of the words or assertions in these rules, not their actual meaning. This is essentially symbol processing rather than knowledge processing. Fuzzy set theory has been utilized to create interface engines that do symbol processing as well as meaning preservation reasoning by associating meaning with linguistic variables in statements.

Efficient approximation: Obtaining precise or ideal answers to practical issues is often unnecessary or undesirable. Getting excellent approximations of a problem's answer fast or with little computing work is often crucial.[6]

1.3 Decision-making in a fuzzy environment

Parameters of a decision-making issue (such as objectives, constraints, achievements, and weights) may all or in part have approximate values. Bellman and Zadeh brought fuzzy set theory to the field of decision-making in order to deal with such ambiguities. A decision, in their view, is the point when one's goals and limitations intersect:[7]

Let

$$\tilde{G}_j = \left\{ (x, \mu_{\tilde{G}_j}(x)) \mid x \in X \right\} \text{ and } \tilde{C}_j = \left\{ (x, \mu_{\tilde{C}_j}(x)) \mid x \in X \right\}$$

be fuzzy sets of objectives ($j = 1, 2, \dots, n$) and constraints ($i = 1, 2, \dots, m$) of a problem in a decision space of X , respectively. A decision is then defined as:

$$\tilde{D} = \left\{ (x, \mu_{\tilde{D}}(x)) \mid x \in X, \mu_{\tilde{D}}(x) = \left[\underset{i}{\otimes} \mu_{\tilde{C}_i}(x) \right] \otimes \left[\underset{j}{\otimes} \mu_{\tilde{G}_j}(x) \right] \right\}$$

Where $\underset{i}{\otimes}, \underset{j}{\otimes}$ and \otimes are aggregation operators.

Fuzzy intersection, as proposed by Bellman and Zadeh, is a way to describe convergence and the combined consideration of objectives and constraints in the context of aggregating operators in 22.

$$D = \tilde{G}_1 \cap \tilde{G}_2 \cap \dots \cap \tilde{G}_n \cap \tilde{C}_1 \cap \tilde{C}_2 \cap \dots \cap \tilde{C}_m$$

and correspondingly form:

$$D = \left\{ \mu_{\tilde{G}_1}(x), \mu_{\tilde{G}_2}(x), \dots, \mu_{\tilde{G}_n}(x), \mu_{\tilde{C}_1}(x), \mu_{\tilde{C}_2}(x), \dots, \mu_{\tilde{C}_m}(x) \right\}$$

They suggested that, depending on the specifics of the situation at hand, the "Min" meaning of the aggregate would need to be adjusted. Decision-makers in a fuzzy setting, therefore, need to develop an acceptable technique to pick the best choice on the basis of aggregated values and a sufficient description for the confluence (aggregation) depending on the context of the problem.

2. LITERATURE REVIEW

Easton A. (2015) Many factors, including investment value, cost, and sales, are unknown or unavailable while making an investment decision, making it an example of a multi-attribute decision making (MADM) problem. The concept of D numbers is a useful tool for dealing with ambiguity and gaps in our understanding. This article examines the idea of interval numbers and D numbers in the context of the unpredictability and incomplete information inherent in financial investment choices. To determine how much emphasis should be placed on various unknown factors, the entropy weight method is used. As a consequence, we provide the MADM model, an original framework for evaluating investment opportunities in light of the D numbers

theory. The effectiveness of the suggested technique is shown using numerical examples.[8]

Atanassov, K. and Gargov, G. (2019) Decision-makers are often given a variety of alternatives from which to choose. How do they evaluate trade-offs when there are more than three criteria? Experts in the area of multiple criteria decision making (MCDM) are continually coming up with new methods for helping people sort through their preferences and assign proper relative weights to various considerations so that they may make the most informed decisions. Contemporary methods of decision making (MADM) are compiled in Multiple Attribute Decision Making: Methods and Applications, with a focus on the fuzzy set method of MADM. The writers rely on their own experiences to construct a wide range of cutting-edge methodology and actual applications of MADM approaches to decision making. They also propose a novel hybrid MADM model that integrates VIKOR processes with those from DEMATEL and ANP.[9]

Fortemps P. and Roubens M. (2016) Optimization is the process of picking a workable solution for the decision maker that meets the required criteria. Using the existing method, decision makers may reach a verdict by selecting many product iterations and comparing their corresponding characteristics. In this research, we discussed two approaches. The grey relations technique is the first way that may be utilized to evaluate a potential replacement. The second method is a ranking fuzzy number-based relative distance. In this study, we provide many interval-based alternatives to grey theory. Ranking fuzzy numbers requires the usage of the triangle membership function. An investment firm looking to capitalize on a novel product type is analyzed in a case study. Five technically feasible possibilities were used to demonstrate the strategy's applicability and effectiveness.[10]

Garg, H. and Tripathi, A. (2017) The difficulty of designing an effective layout is both strategic and highly consequential to the effectiveness of a manufacturing system. Most of the current layout design literature may be locked in a local optimum if it utilizes a surrogate function for flow distance or for simplified goals due to the multiple-attribute decision making (MADM) nature of a layout design choice. As a consequence, the resulting arrangement could be unappealing. This study delves into how MADM methods may be used to solve a difficult design problem in the realm of layout. To illustrate the method, the authors use data from an actual IC packaging company's projects. Fuzzy TOPSIS and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) are proposed as potential solutions to the problem described in the case study. [11]

Jiuying, D. and Ping, W. S. (2016) In traditional multiple attribute decision making (MADM), the simple additive weighting technique (SAW) is the most popular methodology; however, if information is

fuzzy, it is no longer a viable option. Defuzzification is used in the existing approaches of the fuzzy simple additive weighting method (FSAW), which causes fuzzy numbers to be distorted. In addition, the majority of the techniques typically need lengthy and arduous manipulations on the user's part. In this research, we offer a novel fuzzy simple additive weighting approach for use in solving issues involving the use of many attributes in decision-making. In order to prevent defuzzification round off mistakes brought on by multiplication or other arithmetic operations, fuzzy integers are ranked before any fuzzy arithmetic in this approach. This is done to prevent errors from occurring. [12]

3. METHODOLOGY

The Grey Related Analysis methodology

Basic terminology: The use of grey-related analysis may be seen in a variety of different contexts. Apply the idea of the norm of an interval number column vector, as well as the concept of the distance between intervals, product operations, and number-product operations of interval numbers, here.

Let $a=[\bar{a}, \hat{a}]=\{x|\bar{a} \leq x \leq \hat{a}, \bar{a} \leq \hat{a}, \bar{a}, \hat{a} \in R\}$. We call $a=[\bar{a}, \hat{a}]$ a number that is an interval. If $0 \leq x \leq \hat{a}$,

we call interval number $a=[\bar{a}, \hat{a}]$ a number that is positive and an interval. Let $X=([a_1^-, a_1^+], [a_2^-, a_2^+], \dots, [a_n^-, a_n^+])^T$ to take the form of an n-dimensional interval number column vector.

Ranking fuzzy numbers procedure

It would appear that there are benefits and drawbacks associated with using each strategy. A fuzzy number ranking process based on a straightforward mechanism is emphasized here. The following is one possible way to begin with this process:

Intuition ranking method: Many fuzzy numbers may be simply ranked using intuition ranking approach when using membership function curves of fuzzy numbers as the starting point. Lee and Li made the observation that people have a tendency to prefer a fuzzy number that has the properties of a higher mean value while at the same time having a narrower spread.

if there is no way, using the intuitive ranking approach, to rate the ordering of the items. You can rank fuzzy numbers using the -cut technique, the fuzzy mean and spread method, or one of the other available ways. In this case, we do the appropriate rank orderings by utilizing the defuzzification value of the trapezoidal fuzzy number.

The defuzzification value of the trapezoidal fuzzy number

For a trapezoidal fuzzy number $A=(a_1, a_2, a_3, a_4)$, the value that is defined as being its defuzzification:

$$C = \frac{(a_1 + a_2 + a_3 + a_4)}{4}$$

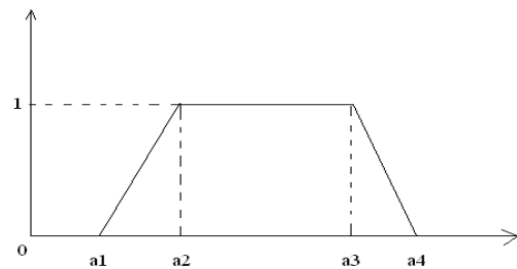


Figure 1.1: Trapezoidal Fuzzy Number

4. RESULTS

The current decision-making environment is very complicated and ambiguous. When making choices in the real world, there is uncertainty about every part of the traditional decision-making paradigm. In multi-attribute models, it may be challenging to deal with precise numerical values owing to the inherent ambiguity and fuzziness of decision-making. We favored a combination of two strategies: grey relational theory and a revised Analytical Hierarchy Process (AHP). Methods that make use of intervals include, but are not limited to, error analysis using interval numbers, linear programming and object programming with feasible regions restricted by interval numbers, and so on

Numerical example

The theoretical approach outlined in this Section has been applied to the material selection for Wind turbine blade. The various alternatives and attributes related to material of wind turbine blade are tabulated below:

Table 4.1: Attribute Alternative

	Stiffness (GPA)	Tensile strength (Mpa)	Density (g/cm ²)	Elongation elongation	Max temp
Steel	30	190	7.5	15	550
Aluminium	10	90	2.7	12	400
Glass-E	73	3500	2.54	3	350
Carbon	350	4000	1.75	1.8	500
Aramid	120	3600	1.45	11	250

Case-I: The grey related analysis

We shall analyze this example with the method of grey related analysis to multiple attribute decision making problems with interval numbers. Assume a multiple attribute decision making problem for selection of materials related to the wind turbine blades and are tabulated as the interval number decision matrix A contains decision maker estimates

of alternative performances on different scales as follows:

Table 4.2: Transform "contrary index" into positive index

Properties Materials	Stiffness (GPA)	Tensile strength (Mpa)	Density (g/cm ³)	Elongationat break (%)	Max temp
Steel	[25,35]	[180,200]	[6, 9]	[10, 20]	[530, 580]
Aluminium	[5, 15]	[80,100]	[1.2,4.2]	[10, 15]	[380, 420]
Glass-E	[67,78]	[3400,3600]	[1, 4]	[1, 5]	[330, 380]
Carbon	[345,355]	[3900,4100]	[0.5, 3]	[1, 2]	[470, 530]
Aramid	[115,125]	[3500,3700]	[0.5,2]	[9, 12]	[220, 280]

The weights w_1, w_2, w_3, w_4, w_5 of attributes G_1, G_2, G_3, G_4, G_5 are uncertain, but the experts can specify the following weight ranges:

$$w_1 \in [0.10, 0.10], w_2 \in [0.20, 0.20], w_3 \in [0.20, 0.20], w_4 \in [0.30, 0.30], w_5 \in [0.40, 0.40].$$

Without loss of generality, we suppose that all the index values are positive.

(1) Standardize the interval number decision matrix A . Let A_1, A_2, A_3, A_4, A_5 denote the close interval column vector of index interval number decision matrix A , respectively, then

$$\|A_1\| = 355, \|A_2\| = 4100, \|A_3\| = 9, \|A_4\| = 20, \|A_5\| = 580.$$

Standardizing the interval number decision matrix converts the initial divergent measures to a common 0-1 scale. Here, we obtain matrix R as follows:

$$R = \begin{bmatrix} [0.0704, 0.0986] & [0.0439, 0.0487] & [0.6666, 1.0000] & [0.5000, 1.0000] & [0.9138, 1.0000] \\ [0.0141, 0.0423] & [0.0195, 0.0244] & [0.1333, 0.4666] & [0.5000, 0.7500] & [0.6552, 0.7241] \\ [0.1887, 0.2197] & [0.8293, 0.8781] & [0.1111, 0.4444] & [0.0500, 0.2500] & [0.5689, 0.6552] \\ [0.9718, 1.0000] & [0.9512, 1.0000] & [0.0555, 0.3333] & [0.0500, 0.1000] & [0.8103, 0.9138] \\ [0.3239, 0.3521] & [0.8536, 0.9024] & [0.0555, 0.2222] & [0.4500, 0.6000] & [0.3793, 0.4827] \end{bmatrix}$$

(2) Calculate the interval number weighted decision matrix C by multiplying the weight intervals by matrix R .

$$C = \begin{bmatrix} [0.0070, 0.0098] & [0.0088, 0.0097] & [0.1333, 0.2000] & [0.1500, 0.3000] & [0.3650, 0.4000] \\ [0.0014, 0.0042] & [0.0039, 0.0048] & [0.0266, 0.0933] & [0.1500, 0.2250] & [0.2621, 0.2896] \\ [0.0188, 0.0219] & [0.1658, 0.1756] & [0.0222, 0.0888] & [0.0150, 0.0750] & [0.2276, 0.2621] \\ [0.0972, 0.1000] & [0.1902, 0.2000] & [0.0111, 0.0666] & [0.0150, 0.0300] & [0.3241, 0.3655] \\ [0.0324, 0.0352] & [0.1707, 0.1805] & [0.0111, 0.0444] & [0.1350, 0.1800] & [0.1517, 0.1931] \end{bmatrix}$$

(3) Determine the reference number sequence U_0 .

$$U_0 = ([0.972, 0.1000], [0.1902, 0.2000], [0.1333, 0.2000], [0.1500, 0.2250], [0.3655, 0.40])$$

(5) Calculate the connection between the sequence composed of weighted interval number standardizing index value of every alternative and reference number sequence.

	G_1	G_2	G_3	G_4	G_5	$\min \square_j(k)$	$\max \square_j(k)$
k							
$\square_1(k)$	0.1804	0.3717	0	0	0	0	0.3717
$\square_2(k)$	0.1926	0.3815	0.2174	0.075	0.2138	0.075	0.3815
$\square_3(k)$	0.1565	0.0488	0.2231	0.36	0.2758	0	0.36
$\square_4(k)$	0	0	0.2556	0.405	0.0759	0	0.405
$\square_5(k)$	0.1296	0.039	0.2778	0.135	0.4207	0	0.4207
$\min \min \square_j(k)$						0	
$i \quad k$							
$\max \max \square_j(k)$							0.4207
$i \quad k$							

$$\text{Let } \Delta_i(k) = \left[\left[u_0^-(k), u_0^+(k) \right] - \left[c_{ik}^-, c_{ik}^+ \right] \right].$$

The connection coefficient $\xi_i(k)$ (a distance function) is then calculated by the formula as follows,

$$\xi_i(k) = \frac{\min_i \min_k \Delta_i(k) + \rho \max_i \max_k \Delta_i(k)}{\Delta_i(k) + \rho \max_i \max_k \Delta_i(k)}$$

In the example, $\rho = 0.5$. When $\xi_i(k)$ is determined $\min_i \min_k \Delta_i(k)$ will be calculated as follows.

From the above table 4.3, we know that $\max_i \max_k \Delta_i(k) = 0.4207$. This is used in the connection coefficient formula to identify distances (larger values means greater distance).

$$\begin{aligned} \xi_1 &= (0.5383, 0.3614, 1.0000, 1.0000, 1.0000) \\ \xi_2 &= (0.5233, 0.3554, 0.4918, 0.7372, 0.4959) \\ \xi_3 &= (0.5734, 0.8117, 0.4853, 0.3688, 0.4327) \\ \xi_4 &= (1.0000, 1.0000, 0.4514, 0.3418, 0.7348) \\ \xi_5 &= (0.6187, 0.8436, 0.4309, 0.6091, 0.3333) \end{aligned}$$

By these results, we know that the connection between every alternative and reference number sequence is, respectively

$$r_1 = 0.7799, r_2 = 0.5187, r_3 = 0.5344, r_4 = 0.7056, r_5 = 0.567$$

Ranking feasible alternatives from largest to smallest r_i the rank order of feasible alternatives is X_1, X_4, X_5, X_3, X_2 .

Case-II: The Improved AHP Methodology

Step 1: Build up hierarchy structure. A_1, A_2, A_3, A_4, A_5 are five selectable material alternatives; they form the alternative hierarchy. Stiffness (GPA) (C_1), tensile strength (Mpa) (C_2), density ((g/cm²)) (C_3),

elongation at break (%) (C_4), maximum temperature (C_5) are five attributes; they form the attribute hierarchy.

Step 2: Construct decision matrix. Normalized decision matrix \tilde{H} is:

Table 4.3 Normalized decision matrix

$$\tilde{H} = \begin{bmatrix} (0.08, 0.08, 0.08, 0.08) & (0.05, 0.05, 0.05, 0.05) & (1, 1, 1, 1) & (1, 1, 1, 1) & (1, 1, 1, 1) \\ (0.03, 0.03, 0.03, 0.03) & (0.02, 0.02, 0.02, 0.02) & (0.36, 0.36, 0.36, 0.36) & (0.8, 0.8, 0.8, 0.8) & (0.73, 0.73, 0.73, 0.73) \\ (0.21, 0.21, 0.21, 0.21) & (0.87, 0.87, 0.87, 0.87) & (0.34, 0.34, 0.34, 0.34) & (0.2, 0.2, 0.2, 0.2) & (0.64, 0.64, 0.64, 0.64) \\ (1, 1, 1, 1) & (1, 1, 1, 1) & (0.23, 0.23, 0.23, 0.23) & (0.12, 0.12, 0.12, 0.12) & (0.91, 0.91, 0.91, 0.91) \\ (0.34, 0.34, 0.34, 0.34) & (0.9, 0.9, 0.9, 0.9) & (0.19, 0.19, 0.19, 0.19) & (0.73, 0.73, 0.73, 0.73) & (0.45, 0.45, 0.45, 0.45) \end{bmatrix}$$

Step 3: Construct 5-dimensional fuzzy attribute evaluation space.

Table 4.4: Construct centrally normalized matrix

$$\tilde{\phi}_{ij}$$

$$\tilde{\phi}_{ij} = \begin{bmatrix} (-0.7182, -0.7182, -0.7182, -0.7182) & (-1.1843, -1.1843, -1.1843, -1.1843) & (1.9518, 1.9518, 1.9518, 1.9518) \\ (-0.8606, -0.8606, -0.8606, -0.8606) & (-1.2528, -1.2528, -1.2528, -1.2528) & (-0.2168, -0.2168, -0.2168, -0.2168) \\ (-0.3476, -0.3476, -0.3476, -0.3476) & (0.6904, 0.6904, 0.6904, 0.6904) & (-0.2846, -0.2846, -0.2846, -0.2846) \\ (1.9036, 1.9036, 1.9036, 1.9036) & (0.9876, 0.9876, 0.9876, 0.9876) & (-0.6235, -0.6235, -0.6235, -0.6235) \\ (0.0227, 0.0227, 0.0227, 0.0227) & (0.7590, 0.7590, 0.7590, 0.7590) & (-0.7929, -0.7929, -0.7929, -0.7929) \\ (1.2385, 1.2385, 1.2385, 1.2385) & (1.3012, 1.3012, 1.3012, 1.3012) \\ (0.6624, 0.6624, 0.6624, 0.6624) & (-0.0819, -0.0819, -0.0819, -0.0819) \\ (-1.0656, -1.0656, -1.0656, -1.0656) & (-0.5430, -0.5430, -0.5430, -0.5430) \\ (-1.2961, -1.2961, -1.2961, -1.2961) & (0.8402, 0.8402, 0.8402, 0.8402) \\ (0.4608, 0.4608, 0.4608, 0.4608) & (-1.1564, -1.1564, -1.1564, -1.1564) \end{bmatrix}$$

Table 4.5: Positive migration for centrally normalized matrix ($\epsilon = 0.01$)

$$\tilde{R} = \begin{bmatrix} (0.1524, 0.1524, 0.1524, 0.1524) & (0.0785, 0.0785, 0.0785, 0.0785) & (2.7547, 2.7547, 2.7547, 2.7547) \\ (0.01, 0.01, 0.01, 0.01) & (0.01, 0.01, 0.01, 0.01) & (0.5861, 0.5861, 0.5861, 0.5861) \\ (0.523, 0.523, 0.523, 0.523) & (0.8256, 0.8256, 0.8256, 0.8256) & (0.5183, 0.5183, 0.5183, 0.5183) \\ (1.053, 1.053, 1.053, 1.053) & (1.1228, 1.1228, 1.1228, 1.1228) & (0.1794, 0.1794, 0.1794, 0.1794) \\ (0.8933, 0.8933, 0.8933, 0.8933) & (0.8943, 0.8943, 0.8943, 0.8943) & (0.01, 0.01, 0.01, 0.01) \\ (2.5446, 2.5446, 2.5446, 2.5446) & (1.3012, 1.3012, 1.3012, 1.3012) \\ (1.9685, 1.9685, 1.9685, 1.9685) & (1.4445, 1.4445, 1.4445, 1.4445) \\ (0.2405, 0.2405, 0.2405, 0.2405) & (0.9834, 0.9834, 0.9834, 0.9834) \\ (0.01, 0.01, 0.01, 0.01) & (2.3666, 2.3666, 2.3666, 2.3666) \\ (1.7669, 1.7669, 1.7669, 1.7669) & (0.01, 0.01, 0.01, 0.01) \end{bmatrix}$$

Table 4.6: Construct five-dimensional fuzzy evaluation matrix

$$\tilde{C\ddot{O}R} = \begin{bmatrix} (2.2636, 2.2636, 2.2636, 2.2636) & (2.4250, 2.4250, 2.4250, 2.4250) & (0.8946, 0.8946, 0.8946, 0.8946) \\ (2.4250, 2.4250, 2.4250, 2.4250) & (2.7489, 2.7489, 2.7489, 2.7489) & (0.8604, 0.8604, 0.8604, 0.8604) \\ (0.8946, 0.8946, 0.8946, 0.8946) & (0.8604, 0.8604, 0.8604, 0.8604) & (8.2328, 8.2328, 8.2328, 8.2328) \\ (2.1222, 2.1222, 2.1222, 2.1222) & (2.0093, 2.0093, 2.0093, 2.0093) & (8.3075, 8.3075, 8.3075, 8.3075) \\ (3.4606, 3.4606, 3.4606, 3.4606) & (3.7132, 3.7132, 3.7132, 3.7132) & (9.5701, 9.5701, 9.5701, 9.5701) \end{bmatrix}$$

$$\begin{bmatrix} (2.1222, 2.1222, 2.1222, 2.1222) & (3.4606, 3.4606, 3.4606, 3.4606) \\ (2.0093, 2.0093, 2.0093, 2.0093) & (3.7132, 3.7132, 3.7132, 3.7132) \\ (8.3075, 8.3075, 8.3075, 8.3075) & (9.5701, 9.5701, 9.5701, 9.5701) \\ (13.5298, 13.5298, 13.5298, 13.5298) & (10.3165, 10.3165, 10.3165, 10.3165) \\ (10.3165, 10.3165, 10.3165, 10.3165) & (16.6498, 16.6498, 16.6498, 16.6498) \end{bmatrix}$$

Step 4: Eigenvector of five-dimensional fuzzy attribute evaluation space. Approximated eigenvector \tilde{W}_i of five-dimensional fuzzy evaluation space $\tilde{C\ddot{O}R}$ can be calculated based on product and root method.

$$\tilde{W}_i = \begin{bmatrix} (0.0998, 0.0998, 0.0998, 0.0998) \\ (0.1037, 0.1037, 0.1037, 0.1037) \\ (0.1698, 0.1698, 0.1698, 0.1698) \\ (0.2682, 0.2682, 0.2682, 0.2682) \\ (0.3585, 0.3585, 0.3585, 0.3585) \end{bmatrix}$$

Step 5: Comprehensive weights \tilde{W}_i of alternative for all attributes.

$$\tilde{W}(A) = \begin{bmatrix} (0.8096, 0.8096, 0.8096, 0.8096, 0.8096) \\ (0.5425, 0.5425, 0.5425, 0.5425, 0.5425) \\ (0.4519, 0.4519, 0.4519, 0.4519, 0.4519) \\ (0.6009, 0.6009, 0.6009, 0.6009, 0.6009) \\ (0.5166, 0.5166, 0.5166, 0.5166, 0.5166) \end{bmatrix}, W(A) = \begin{bmatrix} 0.8096 \\ 0.5425 \\ 0.4519 \\ 0.6009 \\ 0.5166 \end{bmatrix}$$

Step 6: It is obvious that the ranking result is: $A_1 > A_4 > A_2 > A_5 > A_3$.

5. CONCLUSION

The mathematical phenomena represented by fuzzy and intuitionistic fuzzy sets are the subject of this dissertation. We have put in a lot of time and effort and come up with a few new ambiguity measures based on information theory. Additional work has been done to broaden the applicability of aggregation procedures to fuzzy and intuitionistic fuzzy collections. The additional measures offered, which are briefly discussed below, have substantially expanded the scope of the research and supplied tools and methodologies for multiple criterion decision making issues, both of which are in high demand in today's culture.

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