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Inventory Models use of Fuzzy Theory

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Abstract - An essential part of optimising the supply chain is inventory management, but conventional models have a hard time taking into consideration the inherent uncertainties and inaccuracies in real-world data. To make inventory models more accurate and resilient, fuzzy logic—a mathematical framework for dealing with ambiguity and uncertainty—offers a potential solution. Demand forecasting, lead time estimate, and order quantity determination are some of the inventory management components that are examined in this work as they pertain to fuzzy theory integration. There are a lot of reasons that make it hard to anticipate with any degree of accuracy how demand will change for different items. The fuzziness of demand patterns may be captured by using fuzzy demand forecasting approaches, which enable the representation of imprecise information. Our demand forecasting algorithm is built on fuzzy logic and incorporates expert views and historical data to make future demand predictions more accurate and flexible.

Keywords - Inventory, Models, Fuzzy theory

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INTRODUCTION

Elements may be either members of a set or non-members; optimization solutions can be either practical or impractical; Computing and reasoning in formal modelling are also characterised by being crisp, predictable, and exact. Crisp denotes "yes" or "no" type, as opposed to "more" or "less" type. (Ding, H., 2015) The parameters of a model precisely reflect either the characteristics of the actual system being modeled or how the phenomena being modeled is perceived. Typically, when a model is precise, it means it is free of ambiguities, or is very clear.

It shows that when the model's structures and parameters are understood with certainty, there is no room for uncertainty about their values or occurrence. Assumptions made by a formal model, one that does not attempt to mimic reality, are, to some extent, arbitrary; that is, the model builder has considerable leeway to pick and choose which model features to include. (Chang, H. C. 2016) But if the theory or model turns out to be true, then the language used for modeling has to be placed so it can accurately represent the situation's features.

This issue would arise if a modeling language were both non-redundant and clear. Concepts in words are different from the mental processes that give rise to thoughts, visuals, and value systems in humans. (Bylka, S. 2019) Contrasted with rational reasoning, it falls short. As a result, it seems that there is no foolproof way to ensure a direct correspondence between the human mind's conceptualization of a problem and its corresponding mathematical or logical representation. (Chen, S. H. 2015)

Now, let's think about the traits that real-world systems have. In many respects, real-life circumstances are frequently nebulous or unpredictable. (Bylka, S. 2019) The system may not be fully understood since there is a dearth of knowledge about the future. Probability theory and statistics are well-suited to deal with this issue. Nevertheless, the assertions or occurrences are presumed to be well-defined on both the Kolmogorov-type and Koopman's probability approaches. Unlike the "fuzziness" that pertains to the description of the semantic meaning of events, facts, or utterances, this kind of uncertainty or vagueness is stochastically uncertain.. (Kochenberger, G. A 2018)

Areas that rely on human judgment, appraisal, and decision-making often use fuzzy logic. In his everyday speech, human thought is heavily embedded. (Goyal, S. K. 2016) Words are notoriously nebulous in any language. When used as a label for the set, the boundaries inside objects that do or do not belong to the set become fuzzy or imprecise, even if the word's meaning is well-defined. Things like "birds," "tall men," and "beautiful women" go all mushy. In the avian kingdom, bats are an outlier. (Hung-Chi Chang. 2016)

Fuzzy Theory Basics

- **Overview of fuzzy logic**

Mathematically speaking, fuzzy logic is a way of thinking about and making decisions when there is a lot of room for error. Lotfi A. Zadeh first proposed it in the 1960s as a method to simulate human thinking and decision-making under conditions of imperfect and confusing data. (Osman, I. H. 2018)

In fuzzy logic, "fuzziness"—the verbal expression of uncertainty—is the central notion. Fuzzy logic permits values between true and false, as opposed to the binary logic of yes/no, which only accepts absolute zeros and ones. Fuzzy sets and linguistic variables allow us to do this. (Hsu, W. K. 2019)

The following are the main parts of fuzzy logic:

- **Fuzzy Sets:**
  By extending the idea of classical (crisp) sets to include components with partial membership, fuzzy sets broaden their applicability. A membership degree, ranging from 0 to 1, is given to each element to indicate its level of set membership. (Yao, J. S. 2015)
- **Membership Functions:**
  A fuzzy set's degree of membership is defined by a membership function. They determine an element's fit into a certain fuzzy set by mapping input values to membership degrees.
- **Fuzzy Rules:**
  Fuzzy rules are expressions that describe the connections between two variables, one of which is an input and the other an output. To define the actions of the fuzzy logic system, these rules make use of language and if-then expressions. As an example, an air conditioner's output is powerful when the temperature is high. (Yuan, X. M. 2015)
- **Inference System:**
  A fuzzy output is generated by the inference system after processing and combining the fuzzy rules. Fuzzy logic operators like AND, OR, and NOT are used to get the overall result by operating on fuzzy sets.

- **Fuzzy sets and membership functions**

A mathematical framework developed to deal with imprecision and uncertainty in thinking and decision-making, fuzzy logic relies on fuzzy sets and membership functions as its foundational principles. A more effective alternative to conventional binary logic, fuzzy sets were first proposed by Lotfi A. Zadeh in the 1960s and allowed for the representation and manipulation of information that is unclear or ambiguous. (Misra, R. B. 2016)

Fuzzy Sets:

Partially included components may be considered members of fuzzy sets, as opposed to classical sets that only permit inclusion or exclusion. Because of this, items may have membership values in a fuzzy set ranging from 0 to 1, indicating varied degrees of belongingness. The degree of membership shows how strongly an element is associated with the fuzzy set. Because models may be so versatile, they can capture the imprecision and ambiguity that characterize so many real-life situations. (Sosic, G. 2017)

An individual's height, for instance, may have a membership degree of 0.7 in a fuzzy set that represents the category "tall," indicating a high level of affiliation but not total affiliation.

Membership Functions:

The form and properties of fuzzy sets are defined by membership functions, which are vital in fuzzy logic. These functions convert numerical inputs into the degrees to which they belong to a given fuzzy set. Membership functions often take the form of triangles, trapezoids, or even Gaussian curves. (Nagarajan, M., 2017)

Imagine a fuzzy set that stands in for "temperature" and includes words like "cold," "warm," and "hot." These words' membership functions would take the input temperature into account while determining membership degrees. An example of a triangle membership function for the word "warm" would be a peak at a certain temperature followed by a slow drop as the temperature became warmer or cooler.

- **Fuzzy rules and inference systems**

To make decisions and reason in contexts where there is a lot of ambiguity and imprecision, fuzzy logic relies on inference systems and fuzzy rules. For fuzzy input data to become useful and actionable fuzzy output, several components are essential.

Fuzzy Rules:

To represent knowledge in a fuzzy logic system, one must first establish fuzzy rules. Usually using linguistic variables and terminology, they take the shape of conditional assertions. These rules use human-like reasoning to capture the links between input and output variables, reflecting the inherent ambiguity of many real-world events.

"If the temperature is high and the humidity is high, then increase the cooling output." is one possible formulation of a fuzzy rule for an air conditioning system. "High" is a linguistically vague word linked to the specific input variables in this rule. (Park, K. S. 2019).

As an alternative to rigid binary logic, these rules provide a more flexible and human-understandable way for the system to solve problems based on less exact input.

Inference Systems:

By taking the fuzzy rules as inputs and processing them, the inference system may provide a fuzzy output. A complete fuzzy inference is derived by integrating several fuzzy rules. An inference system typically consists of three primary parts: (Hassini, E. 2017)

- **Fuzzification:**
  To use fuzzy logic, one must first determine the membership degrees of the input values using the relevant membership functions. This process transforms the data from crisp to fuzzy sets. The system can now handle language variables and fuzzy logic after this stage.
- **Rule Evaluation:**
  To evaluate a rule, one must apply it to the fuzzified input values using the fuzzy rules and then find out how much of an impact each rule had on the final product. In this step, the antecedents and consequents of the fuzzy rules are combined using fuzzy logic operators like AND, OR, and NOT.
- **Aggregation:**
  A single fuzzy output representing the system's decision or action is produced via aggregation, which combines the multiple rule outputs. Merging the fuzzy sets generated by each rule is a common practice that typically requires operators such as MAX or SUM.

METHODOLOGY

Inventory models play a crucial role in optimizing the balance between supply and demand in various industries. Fuzzy theory, a mathematical framework for dealing with uncertainty, has been applied to enhance traditional inventory models by incorporating imprecision and vagueness in decision-making processes

Data Collection:

Collect secondary data related to the historical demand patterns, lead times, and supplier performance. Ensure the data covers a sufficiently long period to capture variations and uncertainties.

- **Fuzzification of Data:**

Apply fuzzification techniques to convert precise data into fuzzy sets. For example, use linguistic terms to represent demand levels or fuzzy numbers to represent uncertain lead times.

- **Rule Base Development:**

Develop a rule base that describes the relationships between the fuzzy inputs (e.g., fuzzy demand, fuzzy lead time) and the fuzzy outputs (e.g., order quantity, reorder point) based on expert knowledge or historical data patterns.

- **Model Calibration and Validation:**

- Calibrate the fuzzy inventory model using historical data. Adjust the fuzzy sets and rule base parameters to best fit the observed data.
- Validate the model by comparing its predictions with actual outcomes over a different period, ensuring its accuracy and reliability.

RESULTS

Fuzzy Set

Each element in the Universal set is given a value of 1 or 0 by the Crisp set's characteristic function, which distinguishes between elements that are members of the crisp set and those that are not.

A fuzzy set $\widetilde{A}$ is defined by

$$\widetilde{A} = \left\{\left(x, \mu_{\widetilde{A}}(x) / x \in X\right\}, \mu_{\widetilde{A}}(x) \in \left[0, 1\right]\right\},$$

In the classical set A, element x is the first member, and element y is the second member $\mu_{\widetilde{A}}(x)$, to the set of all possible membership grades, which is on the interval [0, 1]. Another definition of the membership function is the extent to which x is compatible with or true in $\widetilde{A}$. As an expansion of the conventional concept of set, Zadeh (1965) created fuzzy sets. In classical set theory, each element may be classified as either belonging to the set or not belonging to the set; in other words, it can be expressed as a binary item. With the use of a membership function with a value in the real unit interval [0, 1], fuzzy set theory enables a progressive evaluation of an element's membership in a set. An expansion of classical set theory, fuzzy set theory allows components to have varying degrees of membership.

Fuzzy number

An improvement over a regular number, a fuzzy number may instead denote a linked collection of potential values, with a weight ranging from 0 to 1, rather than a single, definitive value.

If a fuzzy subset α of R meets the following characteristics, it is referred to be a fuzzy number.

- α is an upper semi-continuous map
- α [a] is nonempty for all a, $0 < a \leq 1$
- α [0] is a bounded subset of R.

Below is the visual depiction:

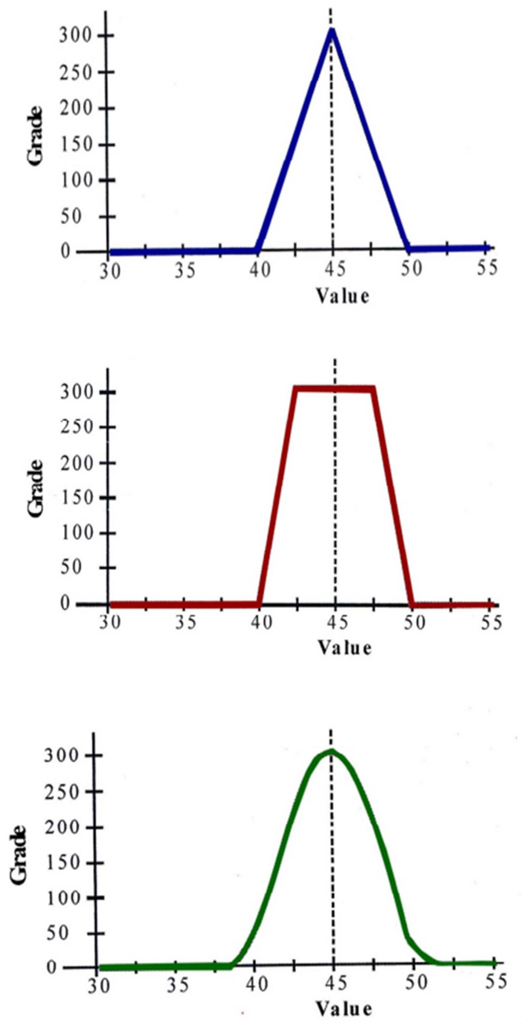

Figure 1: The graphical representation

A triangular fuzzy number is shown by the blue curve in Fig.1.1, a trapezoidal fuzzy number by the red curve, and a bell-shaped fuzzy number by the green curve. The grade begins at 0, grows to a maximum, and then decreases to zero again as the domain expands; all three of these functions are convex, and they are called membership functions. A membership function that is concave, irregular, or chaotic may be associated with a fuzzy number. As long as there is exactly one grade in the range that corresponds to each value in the domain and the grade is between zero and three hundred, the membership curve may take whatever form it wants.

Triangular fuzzy number

A triangular fuzzy number is a specific kind of fuzzy set that stands for numerical values that are uncertain or imprecise. There is some leeway for ambiguity or vagueness when representing a quantity using fuzzy numbers, as opposed to crisp or classical numbers with clear and well-defined values. Fuzzy logic systems often make use of triangle fuzzy numbers because of how easy they are.

The three characteristics that define a triangle fuzzy number are its lower limit (on the left), its modal value (on the center or peak), and its upper bound (on the right). A membership function with three sides is defined by these parameters; it shows how likely it is that a given value falls inside the range.

Triangular fuzzy numbers often have a membership function that looks like a triangle, with the modal value as the point of maximum membership and a linear drop as one approaches the lower and upper boundaries. The points representing the lowest, middle, and highest values are connected to form the triangle.

By definition, a triangular fuzzy number has three possible values: the lower bound ('a'), the modal value ('b'), and the upper bound ('c'). Here is the definition of the membership function Γ(x):

$\mu(x) = \{0, \text{ if } x < a, (x - a) / (b - a),$

if

$a \leq x < b, (c - x) / (c - b),$

if

$b \leq x \leq c, 0, \text{ if } x > c\}$

Here, μ(x) represents the extent to which the value 'x' is a member of the fuzzy set specified by the triangular fuzzy number.

Fuzzy control systems, decision-making, and modeling scenarios involving uncertainty are among the many areas where triangle fuzzy numbers find utility. As a quantitative method, they provide an easy and obvious technique to deal with ambiguity and imprecision.

Trapezoidal fuzzy number

An extension of fuzzy numbers, a trapezoidal fuzzy number provides a more versatile way to express numerical values when there is some degree of uncertainty. Fuzzy logic systems and decision-making processes use trapezoidal fuzzy numbers, which are similar to triangular fuzzy numbers, to describe imprecise or uncertain information. A trapezoidal fuzzy number differs from a triangular fuzzy number primarily in that the membership function takes the form of a trapezium.

In mathematics, the diagonals (a, b, c, d) represent the four corners of a trapezoidal fuzzy number, with 'a representing the left base, 'b' the left shoulder, 'c' the right shoulder, and 'd' the right base. With these parameters, a membership function with a trapezoidal shape can be defined, which measures the likelihood of a value falling inside the given range.

A trapezoidal fuzzy number's membership function μ(x) is defined piecewise using the four parameters:

$$\begin{cases} 0 & \text{if } x \le a \text{ or } x \ge d \\ \frac{x-a}{b-a} & \text{if } a \le x < b \\ 1 & \text{if } b \le x \le c \\ \frac{d-x}{d-c} & \text{if } c < x \le d \end{cases}$$

The degree to which the value 'x' is a member of the fuzzy set described by the trapezoidal fuzzy number is denoted by μ(x) in this formulation.

There is a linear rise from 0 at x≤a to 1 at b in the trapezoidal membership function. The core or modal value of the fuzzy set, stays at 1 on the interval [b,c]. Then, on the interval (c,d), the membership function goes from 1 to 0 linearly. With the shoulders at b and c determining the fuzzy set's breadth, the resultant form resembles a trapezium.

When it comes to representing uncertainty, trapezoidal fuzzy numbers are more flexible than triangular fuzzy numbers. This is because they can accommodate a larger range of values and asymmetric distributions. When dealing with data that is not perfectly accurate or symmetrical, this flexibility becomes quite useful.

Among the many possible uses for trapezoidal fuzzy numbers are fuzzy control systems, decision-making in fuzzy settings, and the simulation of real-world situations involving intrinsically uncertain numerical quantities. They are useful in fuzzy logic and related areas because they can capture a wider range of uncertainty.

Inventory control

Some unforeseen and unexpected results may arise in actual production inventory issues, changing the demand rate of the products and, in turn, the production process schedule, making it more difficult to approach optimality. Consequently, as a result of carrying out various system manufacturing operations, the ideal solution is seldom identified precisely. Therefore, decision-makers have a hard time providing a single, exact figure to more truly reflect the likely and necessary parameters in production inventory issues; therefore, fuzzy numbers are used to address this obstacle. So, it seems that researching and addressing the production inventory issue in various channels is best accomplished using the fuzzy technique. It is more natural and practical.

In production, maintenance, and company operations generally, inventory difficulties are prevalent. Several important expenses, such as those linked with shortages and setup, and demand might be fraught with uncertainty. Classical inventory models use probability theory to deal with uncertainties, which are seen as random events.

Decisions in the construction sector are notoriously difficult to make without extensive training in the many different methods of construction management. Decisions are often imprecise and reliant on specialists' conceptual grasp of the project. Because of this, taking into account information that is both imprecise and unclear is crucial when making a choice. Achieving success in the construction sector depends on the capacity to make optimum decisions in the face of an unpredictable environment. The building project is affected by several aspects, both quantitative and qualitative, including the amount of equipment, the amount of available labor, and weather conditions.

Artificial intelligence methods like expert systems, neural networks, and fuzzy sets may accommodate the uncertainties in the study. Because humans rely so heavily on approximations in everyday thinking, the effective use of fuzzy logic mirrors this reality.

Data certainty, dependability, and accuracy are often illusions in real-world applications. A lot of the data obtained doesn't matter since there aren't many restrictions that determine the best solution to linear programming.

The ability to handle ambiguous and imprecise data might significantly boost the spread and use of linear programming. Using probability distributions for this purpose has not been particularly fruitful. Fuzzy linear programs may be suggested as a way to cut down on information costs without resorting to unrealistic modeling.

Fuzzy inventory models

As a subfield of inventory management, fuzzy inventory models use fuzzy logic to account for data imprecision and uncertainty related to demand, supply, and other variables. Although uncertainties are common in real-world circumstances, traditional inventory models often use parameters with exact and predictable values. To get around this, fuzzy inventory models make use of fuzzy logic and fuzzy sets to describe and manipulate information that isn't perfectly exact.

The fundamental principle of fuzzy inventory models is to represent demand, lead time, and order quantity as fuzzy numbers, which are associated with inventory. Fuzzy numbers provide a way to describe the fuzziness and lack of clarity around certain parameters. Because the input data is inherently inaccurate, the models use fuzzy logic to regulate inventory levels and make judgments.

Fuzzy inventory models often use a fuzzy number to represent demand. Using the parameters (a, b, c) to build a triangle membership function, we may think of a fuzzy demand D.

$$\begin{cases} 0 & \text{if } x < a \text{ or } x > c \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \end{cases}$$

The degree to which the value x is a member of the fuzzy demand set is represented by μD(x) in this equation.

To accommodate for delivery time uncertainties, lead time or order processing time may alternatively be expressed as fuzzy numbers. With a trapezoidal membership function (a, b, c, d), the fuzzy lead time LT is defined as:

$$\begin{cases} 0 & \text{if } x \leq a \text{ or } x \geq d \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{if } c < x \leq d \end{cases}$$

To begin, let's pretend that we're working with a simple fuzzy inventory model. We'll use a triangle membership function to give the order quantity Q a fuzzy value. Taking into account fuzzy demand and fuzzy lead time, the objective is to find the ideal order amount that minimizes overall inventory costs.

Fuzzy numbers may represent the total cost function TC, which takes into account a variety of expenses including holding, ordering, and scarcity:

$$TC(Q) = H(Q) + O(Q) + S(Q)$$

Holding costs (H(Q)), ordering costs (O(Q)), and shortfall costs (S(Q)) are defined here as functions of the fuzzy order quantity Q, respectively.

To maximise TC(Q), the optimal total cost may be achieved by identifying the fuzzy order quantity ∏Q∏. When uncertainty is a major factor in inventory management, this optimization approach is appropriate because it takes the imprecision of demand, lead time, and order amount into account.

By taking into account the uncertainties that exist in actual supply chain systems, fuzzy inventory models provide a more practical and adaptable method of inventory management. They provide a solid foundation for making choices with imperfect data, which in turn allows for more flexible and responsive inventory management solutions.

CONCLUSION

When dealing with the inherent uncertainties in supply chain and inventory management, including fuzzy theory in inventory models offers a more realistic and flexible approach. It helps decision-makers to better account for demand fluctuation, lead times, and other essential elements while recognizing the imperfect nature of data. More reliable and adaptable inventory management systems are the result of better decision-making processes made possible by using fuzzy logic in inventory models. Companies thrive in unpredictable and ever-changing markets because fuzzy theory improves their capacity to adjust to new circumstances, which in turn boosts performance and delights customers.

REFERENCES

1. Ding, H., Benyoucef, L., & Xie, X. (2015). A Simulation Optimization methodology for supplier selection problem. International Journal of Computer Integrated Manufacturing, 18(2-3), 210-224.
2. Aissani, N., Haouari, M., & Hassini, E. (2017). Supplier selection and order lot sizing modeling: A review. Computers and Operations Research, 34, 3516-3540.
3. Chang, H. C. (2016). An application of fuzzy sets theory to the EOQ model imperfect quality items. Computers and Operations Research, 31(12), 2079-2092.
4. Biswajit Sarkar, Arunava Majumder. (2018). Integrated vendor-buyer supply chain model with vendor's set up cost reduction. Applied Mathematics and Computation, 224, 362-371.
5. Bylka, S. (2019). Competitive and Co-operative policies for the vendor–buyer system. International Journal of Production Economics.
6. Chen, S. H. (2015). Operations on fuzzy numbers with function principle. Tamkang Journal of Management Sciences, 6(1), 13-25.
7. Kochenberger, G. A. (2018). Inventory models, optimization by geometric programming. Decision Sciences, 2(1971), 193-205.
8. Glock, C. H. (2016). Supply Chain coordination Via integrated inventory models: A review, working paper, University of Wuerzburg.
9. Goyal, S. K. (2016). An integrated inventory model for a single supplier–single customer problem. International Journal of Production Research, 5, 107-111.
10. Hill, R. M. (1997). The Single-Vendor Single-buyer Integrated production inventory model with a generalized policy. European Journal of Operations Research, 97, 493-499.
11. Hung-Chi Chang. (2016). An application of fuzzy sets theory to the EOQ model with imperfect quality items. Computers and Operations Research, 31, 2079-2092.
12. Jaber, M. Y., & Osman, I. H. (2018). Coordinating a two-level supply chain with delay in payments and profit-sharing. Computers and Industrial Engineering, 50, 385-400.
13. Kao, C., & Hsu, W. K. (2019). A single-period inventory model with fuzzy demand.

Computers and Mathematics with Applications, 43, 841-898.
14. Lin, D. C., & Yao, J. S. (2015). Fuzzy economic production for production inventory. Fuzzy Sets and Systems, 111, 465-495.
15. Liu, L., & Yuan, X. M. (2015). Coordinated replenishment in inventory systems with correlated demands. European Journal of Operational Research, 123(3), 490-503.
16. Misra, R. B. (2016). Optimal production lot size model for a system with deteriorating inventory. International Journal of Production Research, 15, 495.
17. Nagarajan, M., & Sosic, G. (2017). Conditions Stability in Assembly Models, 53, Operations Research, 57, 131-145.
18. Park, K. S. (2019). Fuzzy theoretic interpretation of economic order quantity. IEEE Transactions on Systems, Man, and Cybernetics, 1082-1084.

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