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REVIEW ARTICLE

STUDY ON HYPERGRAPHS AND DIRECTED HYPERGRAPHS

Study on Hypergraphs and Directed Hypergraphs

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Abstract – A graph is often thought of as an abstract structure that represents the pairwise connections between collections of objects known as vertices. Two vertices may be linked by an edge or can exist independently of one another. Allowing an edge to link an arbitrary number of vertices is one method to broaden this notion. Hyperedges are subsets of the vertex set, and they are referred to as such. A hypergraph is made up of a set of vertices and a family of hyperedges. Hypergraphs are more abstract than graphs, having less structure. Hypergraphs, rather than graphs, are better suited as a modelling paradigm in certain situations. Hypergraphs are used to simulate tram lines in [Karbstein, 2012]) and railway vehicle coupling in [Borndörfer et al., 2012] in the area of transportation planning. See [Eiter and Gottlob, 1995] for examples of hypergraphs in the fields of logic and artificial intelligence. In addition, both directed and undirected hypergraphs are effectively employed in the area of biological networks analysis; for a brief review, see [Klamt et al., 2009]. Protein interactions, for example, often include more than one protein, therefore hyperedges rather than edges may be utilised to simulate them more correctly.

Keywords – Hypergraphs, Directed

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INTRODUCTION

Hypergraphs, a generalization of graphs, were broadly and deeply studied in Berge (1973,1984, 1989), and quite often have proved to be a a hit device to represent and model concepts and structures in diverse areas of Computer Science and Discrete Mathematics. The advancement of graph theory paved the way for discovering answers to real-world issues including finding the shortest route, reducing costs, and scheduling in numerous sectors, among others. Many networking issues benefit from graph theoretical techniques. Hyper graphs are graphs that have been generalised. The key conceptual distinction between standard graph and hypergraph theory is that in a graph, a particular edge connects two nodes, but in a hypergraph, the so-called hyper-edges may connect more than two vertices. Hypergraph's more flexible representational approach led to new concepts in a variety of disciplines of computer science and informatics requiring large-scale composite systems. As a result, hypergraphs have gained new application areas and momentum.

Definition The dual of a hypergraph $\mathcal{H} = (X, \mathcal{E})$ where $X = \{X_1, X_2, \ldots, X_n\}$ and $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$ is a hypergraph H^* with vertex set $\mathcal{E}$ and edge set $\{X_1, X_2, \ldots, X_n\}$ where $X_i = \{E_j : X_i \in E_j\}$.

Definition For any hypergraph $\mathcal{H} = (X, \mathcal{E})$, two vertices v and ω are adjacent if there exists an edge $E \in \varepsilon$ that contains both v and ω, Otherwise, they are not adjacent. In graphs, it's the same as supposing that a set doesn't have any edges or that any two of its vertices aren't near. However, in hypergraphs, the first requirement is weaker than the second.

HYPERGRAPHS

Graphs are considered with the possibility of loops and parallel edges, and the same approach is used for hypergraphs. A hypergraph is denoted as $H = (V, \varepsilon)$, where V is the set of nodes and ε is the set of hyper-edges. Hyper-edges are considered to be multisets. To a hyper-edge e we associate the characteristic function $\chi_e : V \to \mathbb{Z}_+$, i.e $\chi_e(v)$ equals the node v's multiplicity in the hyper-edge e. When discussing the relationship between a hyper-edge e and a set X, several specific notations will be used:

$v \in e$ means that $\chi_e(v) > 0$, $|e| = \chi_e(v)$, $|e \cap X| = \chi_e(X)$, $e \subseteq X$ means that $\chi_e(V - X) = 0$, The hyper-edge $e \cap X$ is defined as $\chi_{e \cup X}(v) := \chi_e(v) * \chi_x(v)$,

The hyper-edge $e - X$ is defined as $e \cup (V - X)$,

The hyper-edge $e_1 - e_2$ is defined as $\chi_{e_1 - e_2}(v) := \left(\chi_{e_1}(v) - \chi_{e_2}(v)\right)^+$.

For $v \in V$, the hyperedge $e + v$ is defined as $\chi_{e+v} := \chi_e + \chi_{\{v\}}$,

For a hyper-edge set ε', $\cup(\varepsilon')$ is the smallest subset X of V for which $e \subseteq X$ for every $e \in \varepsilon'$

A –hyper-edge is a hyper-edge e with $|e| =$ for a positive integer v. A hypergraph is -uniform if every hyperedge has the same cardinality. The cardinality of a hypergraph's greatest hyper-edge is the rank of the hypergraph. A hyper-edge e (hyper-edge e) is a kind of edge that $H = (V, \varepsilon)$ is induced by a subset X of V if e ⊆ X. The number of hyper-edges induced by X is denoted by $i_H(X)$. The degree of a node $v \in V$ is $d_H(v) := \Sigma_{e \in \varepsilon}\, \chi_e(v)$.

In a hypergraph H, a path between nodes s and t is an alternating sequence of distinct nodes and hyper-edges $s = v_0, e_1, v_1, e_2, \ldots\ldots, e_k, v_k = t$, such that $v_{i-1}, v_i \in e_i$, for all $i = 1, \ldots, k$. Figure 1.1 shows an example of a path between two nodes. H is connected if there is a path between any two distinct nodes. A hyperedge e enters a set X if $e \cap X \neq \phi$ and $e \cap (V - X) \neq \phi$. It is easy to see that H is connected if and only if every non-empty proper subset of V is entered by at least one hyper-edge of H.

For a hypergraph $H = (V, \varepsilon)$, we define, $\Delta_H(X) = \{e \in \varepsilon : e \text{ enters } X\}$ and $d_H(X) := |\Delta_H(X)|$. Note that $d_H(\{v\}) \text{ and } d_H(v)$ because hyper-edges are multisets, they might be distinct. In the case of subsets $X, Y \subseteq V$ let $d_H(X, Y)$ be the number of hyper-edges $e \in \varepsilon$ with e ⊆ X ∪ Y, $e \nsubseteq X, e \nsubseteq Y$. Every hypergraph has the following properties:

$$d_H(X) + d_H(Y) \geq d_H(X \cap Y) + d_H(X \cup Y) \quad for\ every\ X, Y \subseteq V \quad \ldots\ldots\ldots(1.1)$$

$$i_H(X) + i_H(Y) = i_H(X \cap Y) + i_H(X \cup Y) - d_H(X, Y) \quad for\ every\ X, Y \subseteq V \ \ldots\ldots(1.2)$$

It is well known that Theorem 1.1 of Menger can be generalized for hypergraphs:

Theorem Let $H = (V, \varepsilon)$ be a hypergraph with different nodes s, t, and V Between s and t, the maximum number of edge-disjoint pathways is

$$\min\{d_H(X) : X \subseteq V \ \ is\ an\ \ \bar{s}t - set\}.$$

As for graphs $\lambda_H(s, t)$ The local edge-connectivity between s and t is defined as the greatest number of edge-disjoint pathways between s and t. If k is a positive integer, then a hypergraph $H = (V, \varepsilon)$, If the following comparable criteria apply, it is referred to as k-edge-connected:

(i) $\lambda_H(u, v) \geq k$, for every pair $u, v \in V$ of distinct nodes.

(ii) $d_H(X) \geq k$, holds for every non-empty proper subset X of V .

(iii) To break down H into two halves, at least k hyper-edges must be removed.

(iv) If we remove k 1 hyper-edge from H, it stays linked. If $d_H(X)\ p(X)$ for all X V, a hypergraph H is said to cover a set function p. As a result, if we define the set function pk as follows:

$$p_k(X) := \begin{cases} k & if \quad \phi \neq X \subset V, \\ 0 & if \quad X = \phi \quad or \quad X = V, \end{cases}$$

, then H is k-edge-connected if and only if it covers p_k.

Directed Hypergraphs

A directed hypergraph is defined as a pair $D = (V, A)$, where V denotes a finite ground set and A denotes a finite collection of so-called hyperarcs (possibly with repetition). A hyperarc an is a hyper-edge (which we will also refer to as an if this creates any confusion) with a predefined head node $h(a)$, a and the remainder of its nodes marked by t. (a). As a result, the function of head and tails is asymmetric: $h(a)$ is a node, but t(a) is a multiset. A natural way to think about a directed hypergraph is as a hypergraph's orientation. $H = (V, \varepsilon)$, i.e., a head node $h(e) \in e$ is designated for every hyperedge $e \in E$. The underlying hypergraph of a directed hypergraph D is the one obtained by considering each hyperarc as a hyperedge.

An $(r,1)-$hyperarc is a hyperarc that has r tail-nodes. If every hyperarc is a $(r,1)$-hyperarc, $D = (V,A)$ is $(r,1)$-uniform. If an X, a hyperarc an of D is produced by a subset $X\ V$. The number of D hyperarcs caused by X is given by $i_D(X)$. The indegree of a node $v \in V$ is $\varrho_H(v) = |\{a \in A : h(a) = v\}|$. The out-degree of $v \in V$ is $\delta_H(v) = \Sigma_{a\in A}\ \chi_{t(a)}(v)$.

A hyperarc a enters a set $X \subseteq V\ if\ h(a) \in X$ and $t(a) \nsubseteq X$. For a directed hypergraph $D = (V,A)$ we define

$$\Delta_D^-(X) = \{a \in A :\ \ a\ enters\ X\},$$

$$\Delta_D^+(X) = \{a \in A :\ \ a\ enters\ V - X\},$$

$\varrho_D(X) := |\Delta_D^-(X)|$, and $\delta_D(X) = |\Delta_D^+(X)|$.

For subsets $X, Y \subseteq V$ let $d_D(X,Y)$ be the number of hyperarcs $a \in A$ with, $a \subseteq X \cup Y$, $a \nsubseteq X$, $a \nsubseteq Y$. The following is true for every directed hypergraph D and subsets $, Y \subseteq V$:

$$\varrho_D(X) + \varrho_D(Y) = \varrho_D(X \cap Y) + \varrho_D(X \cup Y) + d_D(X,Y) \qquad (1.4)$$

$$\varrho_D(X) + \varrho_D(Y) = \varrho_D(X \cap Y) + \varrho_D(X \cup Y) + d_D(V-X, V-Y) \qquad (1.5)$$

Theorem 1.2 Extends naturally to directed hypergraphs:

Proposition 1.1 In a directed hypergraph $D = (V,A)$, there exist k edge-disjoint paths from node s to node t if and only if $\varrho_D(X) \geq k$ for every $\bar{s}t - set\ X$.

Proof. Suppose that $\varrho_D(X) \geq k$ for every $\bar{s}t - set\ X \subseteq V$. To reduce the problem to the digraph case, a new node v_a is added to V for every hyperarc $a \in A$, and the hyperarc a is replaced by edges uv_a for every $u \in t(a)$, and an edge $v_a h(a);\ let\ D' = (V', A')$ be the obtained digraph. There is a one-to-one correspondence between the paths from s to t in D and the paths from s to t in D', and the disjointness of the edges is kept The largest number of edge-disjoint pathways from s to t, according to Theorem 1.2, is $\min\{\varrho_{D'}(X') : X'\ \ is\ an\ \ \bar{s}t - set\ in\ V'\}$.

for such an X', let $X := X' \cup V$; then $k \leq \varrho_D(X) \leq \varrho_{D'}(X')$.

As a result, local edge-connectivity may be defined in the same way as digraphs: for nodes that are distinct $s,t \in V,\ \lambda_D(s,t)$, From s to t, is the maximum number of edge-disjoint pathways. In terms of global connectedness, the following is true:

Proposition 1.2. For a directed hypergraph $D = (V,A)$ and a positive integer k, the following are equivalent:

(i) $\lambda_D(u,v) \geq k$, for every pair $u,v \in V$ of distinct nodes.

(ii) $\varrho_D(X) \geq k$ Holds for every non-empty proper subset X of V.

(iii) D remains strongly connected if we delete any $k-1$ edges.

A hypergraph that is directed, if the above holds for D, D is referred to be k-edge-connected. If D is k-edge-connected from S to T given S, T, and V, it is said to be k-edge-connected from S to T, $\lambda_D(s,t) \geq k$ for every distinct $s \in S$ and $t \in T$.

OBJECTIVES

1. To learn more about fuzzy hypergraphs and intuitive fuzzy hypergraphs.

2. To use intuitionistic fuzzy hypergraph knowledge in real-world applications.

REVIEW OF LITERATURE

Berge [2008] developed the notion of hypergraphs, which has since been regarded as a valuable tool for analysing system structure and representing partitioning, covering, and clustering.

Although hypergraphs are not as prevalent as graphs, they do appear in a variety of applications. Database schemata and hypergraphs have a natural correspondence in relational databases, with characteristics matching to vertices and relations to hyper-edges. Hypergraphs are used in VLSI design to visualise circuits, as well as in computational biology and social networks. Directed hypergraphs (Ausiello et al.,; Gallo et al.,) are an extension of directed graphs (digraphs) that may represent binary relationships between subsets of a given set. Database systems (Ausiello et al.), expert systems (Ramaswamy et al.), parallel programming (Nguyen et al.)[74], Scheduling (Lin and Sarra fzadeh, Gallo and Scutella), routing in dynamic networks (Pretolani), data mining (Chawla et al.) and bio informatics all have similar linkages (Krishnamurthy et al.).

Gallo et aldefinitions .'s of directed hypergraph subfamilies may be linked to older definitions such as the one offered by Ausilo et al. B-graph, F-graph, and BF-graph are examples of subfamilies. A digraph is an example of a BF-graph.

A hypergraph's visual depiction is just as significant as that of graphs and digraphs. Makinen proposed the subset standard and the edge standard, two hypergraph drawing concepts based on techniques for defining hypergraphs. The first one takes use of the fact that a hypergraph is a set of subsets that can be shown as a Venn diagram. With this standard, Bertault and Eades [2012] proposed a drawing system that concentrates on the depiction of hypergraphs. A hyperedge e is represented in the edge standard by connecting the points that represent the vertices that form e with curve lines that must cross at a single place, emphasising the image of a single edge. For directed hypergraphs, the edge standard is the ideal option since the hyper-edges may be drawn as two sets linked by lines. In fact, practically every study on the topic has adopted this graphic portrayal.

Because hyper-edges naturally give a representation of implication dependencies, directed hypergraphs have a wide range of uses. Among other things, they were used to answer a number of difficulties in propositional logic relating to satisfiability, particularly in relation to Horn formulae. They also show up in issues involving network checking, chemical reaction networks, and, more recently, convex polyhedral algorithmic in tropical algebra. Many algorithmic elements of directed hypergraphs related to optimization have been explored, including calculating shortest pathways, maximum flows, least cardinality cuts, and minimal weighted hyperpaths. None of the directed graph techniques can be extended to directed hypergraphs, unfortunately. The fundamental reason for this is because hypergraphs' reachability relations do not have the same structure.

CONCLUSION

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