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A Study on Generalized Real Analysis and Its Applications

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Abstract – In this paper there are emphasized some of the benefits of a simplified real analysis (called pseudo-analysis) based on some real operations which are taken instead of the normal addition and product of reals. Namely, there are covered by one philosophy and so with centralized approaches several issues (usually nonlinear) from several fields (system theory, optimization, control theory, differential equations, difference equations, etc.). There are provided several essential actual aggregation functions as triangular norms and triangular conorms and a real semiring with pseudo-operations. First it is presented how these operations arise as simple operations in the philosophy of fuzzy logics and fuzzy sets and there is seen a generalization of the utility theory represented by hybrid probabilistic – possibilistic calculation. The real semirings serve as a base for pseudo-additive measures, pseudo-integrals, pseudo-convolutions which shape the pseudo-analysis. There are provided some of the implementations by broad variance theorem, nonlinear Hamilton – Jacobi equation, cumulative prospect theory.

Keywords: Real Analysis, Aggregation Functions, Pseudo-Analysis and Generalizations, Applications;

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INTRODUCTION

In reality, constructing models frequently face nonlinearity and complexity, and we aim to find the best model. We typically have to use various mathematical methods for this reason. We will present some findings here for a widespread mathematical study known as pseudo-analysis. A semiring at a real interval is taken for the set of functions and measurements instead of the area of real numbers $[a,b] \subset [-\infty,+\infty]$; denotes the related activities as $\oplus$ (pseudo-addition) and $\odot$ (pseudo-multiplication). Modeling nonlinearity, instability in many modelling questions, nonlinear partial differential equations, nonlinear differential equations, optimal control, fuzzy systems, game- theory are many different implementations of this principle.

From the seed framework, Maslov and his party, a so-called ide powerful (with some traces before in some application), establish the so-called pseudo-analytic approach in a more general sense analogously, as the classical analysis, implementation of pseudo-additive steps, pseudo-integral, pseudo-convolution, transformation of pseudo-Laplace etc. This framework is used to solve nonlinear equations (ODE, PDE, Differential equations) using the pseudolinear overlay concept which often offers a pseudolinear mix of nonlinear equation solutions. The benefit of pseudo-analysis is that questions (usually non-linear and uncertain) in several separate areas are covered by one principle and so by coherent approaches. It is essential because this method provides solutions in the form that other hypotheses, such as the Bellman differential equation, Hamilton-Jacobi equations with non-smooth Hamiltonians, are not achieving. It is also essential to analyses the fundamental actual operations with different aggregation functions as well as non-additional measures.

In the next segment we introduce first general aggregation and then some unique real-world operations, namely triangular norms and triangular conorms, and then a real pseudo-operated seedling. In Section 3, we demonstrate briefly that previous operations exist in the theory of fuzzy logics and fuzzy devices as fundamental operations and present a generalization of utility theories as hybrid probabilistic-possibilities measure. In the fourth section we present some fundamental definition of the pseudo-analysis as a pseudo-additive measure, pseudo-integral and pseudo-convolution with particular emphasis on idempotent analysis with the basic theorem of representation and its achievement through the restricted procedure per case developed. demonstrates several pseudo-analysis applications of the theory of broad variance theorem, the Hamilton-Jacobi nonlinear equation, cumulative theory of prospects.

WHAT IS ANALYSIS?

The word 'analysis' now encompasses vast portions of algebra. In order to grasp what it entails, you almost have to be a trained mathematician.

In a course of this degree, though, the "real analysis" primarily applies to the subject matter you have already learned in your calculus courses: limits, consistency, derivatives, integrals and sequences and series. The calculus as a subject may be considered as an invention of the eighteenth century, the analysis as a production of the nineteenth century.

The basis of the topic was reworked in the first decades of the nineteenth century, primarily by Cauchy (whose name is sometimes included in this text), and new and effective methods were developed.

We shall again look at series limits, function limits etc, as we saw previously in our calculus courses. We want to know just what they say and how to show the relevance of the subject techniques.

At first glance, you may wonder. We're only checking our estimate, but now, we can't miss the specifics of the evidence? However, once you persist in doing so, you can notice that we are in the place of a modern and unfamiliar universe. By looking carefully into the specifics of why these processes function, we are getting a new perspective into them.

REAL OPERATIONS

Aggregation functions

What is Aggregate Function?

An aggregate function is a mathematical equation with a series of values that contribute to a single value that shows the importance of the data from which it is measured. In databases, spreadsheets and several other programme programmes for data manipulation aggregate functions are popular in the workplace. In the financial sense, aggregate roles in economy and finance are commonly used to include main figures reflecting economic health, stock and output in the sector.

Understanding Aggregate Function

The aggregate function simply refers to the calculations carried out on a data set to obtain a single number which accurately shows the underlying data. Computers also increased the efficiency of these measurements, enabling aggregate functions to generate results easily and also change weights depending on the trust the consumer has in the details. Thanks to machines, aggregate functions can accommodate exponentially broad and dynamic data sets.

Popular add-on functions include:

- Decent (also known as average arithmetic)
- Count
- Maximum
- Nan mean (a mean that NaN values are overlooked, often referred to as zero or null)
- Marginal Median
- Minimum
- Mode
- Sum

In several applications, we shall commence operations at the real interval [0, 1] and an essential general class of operations. Adding several input values to the single output value is an important mathematics instrument with many implementations in physics, architecture, economics, social sciences, and other sciences.

Definition 1. A *n-ary* group structure is a function $\mathbf{A}^{(n)}: [0,1]^n \to \mathbb{R}$ This in each position is not diminishing and satisfies the following limit criteria

$$\inf_{(x_1,\ldots,x_n)\in[0,1]^n} \mathbf{A}^{(n)}(x_1,\ldots,x_n) = 0 \quad \text{and} \quad \sup_{(x_1,\ldots,x_n)\in[0,1]^n} \mathbf{A}^{(n)}(x_1,\ldots,x_n) = 1.$$

Definition 2. An expanded aggregation function is a sequence $(\mathbf{A}^{(n)})_{n\geqslant 1}$, whose n_{th} member is a feature of *n-ary* aggregation $\mathbf{A}^{(n)}: [0,1]^n \to \mathbb{R}$.

We note that in basic, the general extended aggregation functions A(n) and A(m) for different n and m do not have to be connected. Some features, including associativity or decomposability, nevertheless push these relationships. In several implementations we typically need certain extra functions of the aggregation operator, which render mathematical computation simpler on the one hand and better suits the model specifications on the other.

Triangular norms and conorms

We have a specific essential connective aggregation function, i.e. $\mathbf{A} \leqslant \mathrm{Min}$. The effects of such functions will only be large if all input values are high.

Definition 3. A triangular regular T (briefly *t-norm*) is a function $\mathbf{T}: [0,1]^2 \to [0,1]$ such that

(T1)	$\mathbf{T}(x,y) = \mathbf{T}(y,x)$	(commutativity)
(T2)	$\mathbf{T}(x, T(y,z)) = \mathbf{T}(\mathbf{T}(x,y),z)$	(associativity)
(T3)	$\mathbf{T}(x,y) \leqslant \mathbf{T}(x,z)$ for $y \leqslant z$	(monotonicity)
(T4)	$\mathbf{T}(x,1) = x$	(boundary condition)

The foregoing are the key *t-norms*

$$\mathbf{T_M}(x,y) = \min(x,y), \quad \mathbf{T_P}(x,y) = xy,$$

$$\mathbf{T_L}(x,y) = \max(0, x+y-1), \quad \mathbf{T_D}(x,y) = \begin{cases} \min(x,y) & \text{if } \max(x,y) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

The required dual activity, i.e. a breakdown aggregation feature, $\mathbf{A} \leqslant \mathbf{Max},$, is generated by

Definition 4. A triangular co-norm S (shortly *t-conorm*) is a function $\mathbf{S} : [0,1]^2 \to [0,1]$ such that

(S1)	$\mathbf{S}(x,y) = \mathbf{S}(y,x)$	(commutativity)
(S2)	$\mathbf{S}(x,\mathbf{S}(y,z)) = \mathbf{S}(\mathbf{S}(x,y),z)$	(associativity)
(S3)	$\mathbf{S}(x,y) \leqslant \mathbf{S}(x,z)$ for $y \leqslant z$	(monotonicity)
(S4)	$\mathbf{S}(x,0) = x$	(boundary condition)

The following are the most important t-conorms:

$$\mathbf{S_M}(x,y) = \max(x,y) \,, \quad \mathbf{S_P}(x,y) = x + y - xy,$$

$$\mathbf{S_L}(x,y) = \min(1, x+y), \quad \mathbf{S_D}(x,y) = \begin{cases} \max(x,y) & \text{if } \min(x,y) = 0 \\ 1 & \text{otherwise.} \end{cases}$$

There are several other main t-norms and t-conorms in.

Applications of t-norms and t-conorms

t-Norms in fuzzy logics and fuzzy sets

We can quite quickly demonstrate where t-norms and t-conorms exist in their fuzzy logics and fuzzy sets. For information and for t-norms and t-conorms, respectively, separate from min and peak. Taking a t-norm T, the efficient Zadeh negation c $c(x) = 1 - x$ and, indirectly, the t-conorm S dual to T given by $\mathbf{S}(x,y) = c(\mathbf{T}(c(x), c(y)))$, We should add the fundamental connectives in a $[0,1]$ Precious logic as follows

$$\text{conjunction}: \quad x \wedge_{\mathbf{T}} y = \mathbf{T}(x,y), \qquad \text{disjunction}: \quad x \vee_{\mathbf{T}} y = \mathbf{S}(x,y).$$

If x and y are the real interests of two procedures A and B, so x ^ Ty is the true value of "*A AND B*"' $x \vee_{\mathbf{T}} y$ Is the actual '*A OR B*' value and c(x) is the real '*NOT A*' value. As we limit ourselves to Boolean logic (i.e. two-values) with only truth values 0 and 1, the classical logical connectives are received. Additionally, $([0,1], \mathbf{T}, \mathbf{S}, c, 0, 1)$ The Boolean algebra never yields. As in classical logic, inference, two-way involvement, etc. may be built by negation, conjunction and breakup. Considering that '*NOT A OR B*' is equal to '*IF A THEN B*' in Boolean logic, one way of modelling the inference in a [0, 1]-assessed logic is to describe the function. $I_{\mathbf{T}} : [0,1]^2 \to [0,1]$

$$I_{\mathbf{T}}(x,y) = \mathbf{S}(c(x), y) = c(\mathbf{T}(x, c(y)))$$

It is obvious that the contraposition rule in this case $I_{\mathbf{T}}(x,y) = I_{\mathbf{T}}(c(y), c(x))$) is still true. Is always correct. For TM; TL, the following implications are accomplished:

$$I_{\mathbf{T_M}}(x,y) = \begin{cases} y & \text{if } x + y \geqslant 1, \\ 1 - x & \text{otherwise,} \end{cases} \qquad I_{\mathbf{T_L}}(x,y) = \begin{cases} 1 & \text{if } x \leqslant y, \\ 1 - x + y & \text{otherwise.} \end{cases}$$

Another option to expand the classical binary involvement operator to the interval of the unit [0, 1] uses the residue

$$R_{\mathbf{T}}(x,y) = \sup\{z \in [0,1] \mid \mathbf{T}(x,z) \leqslant y\}.$$

TL obtains the following residues for the two previously relevant t-norms TM:

IT and RT are essentially different (though they are Boolean extensions), but they are different. $I_{\mathbf{T_L}} = R_{\mathbf{T_L}}$.

Provided the (crisp) universe of speech *X*, a fuzzy subset of subset A of *X* is well recognised for its membership function $\mu_A : X \to [0,1],$, where for $x \in X$ the number $\mu_A(x)$ The level of membership of x in the fuzzy set A, or equivocally the true value of the expression 'x is the feature of A' is interpreted. Participation Membership μ_A of a fuzzy subset A of *X*, the characteristic function is a very natural generalization $\mathbf{1}_B : X \to \{0,1\}$ of a crisp subset of *X*, the value is given 1 to all *X* elements belonging to B, and 0 to all other *X* elements. It is very natural to use triangular norms and conorms in order to generalize Boolean set logical operations, such as intersection and union (or, in similar words, the analogous logical operations conjunction and disjunction). The transfer function of intersection A \ B, union A [B, and supplement Ac shall be given by the t-norm T and t-conorm S for any fuzzy subsets A and B of universe *X* $A \cap B,$ the union $A \cup B$ and the addition Ac is given by

$$\mu_{A \cap B}(x) = \mathbf{T}(\mu_A(x), \mu_B(x)), \quad \mu_{A \cup B}(x) = \mathbf{S}(\mu_A(x), \mu_B(x)),$$

$$\mu_{A^c}(x) = 1 - \mu_A(x).$$

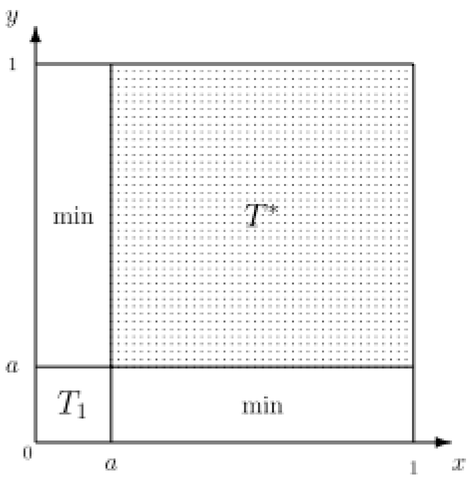
The values $\mu_{A \cap B}(x), \mu_{A \cup B}(x)$ and $\mu_{A^c}(x)$ Describe truth values for claims '*x* is the *AND x* element is *B* element,' '*x* is an *AOR x* element is *B* element,' and '*x is NOT* the *A* part, Let $X_1 \times X_2$ Crisp cartesian product of two *X1 and X2* crisp packages. Then a flippant partnership R $X_1 \times X_2$ is a fuzzy set on $X_1 \times X_2$. In various applications, such as the automatic control by fuzzy controllers, the definition of the fuszy relation equation, first introduced in for TM, is significant. For observations on a solution to a specific form of fluid relationship equation.

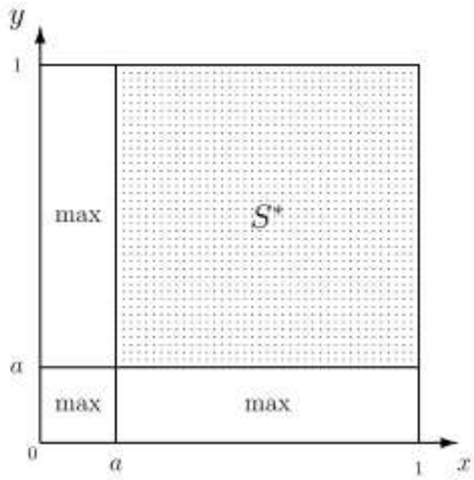
Hybrid utility function

T is distributive over S provisionally, i.e. they obey the property (CD):

$$\mathbf{T}(x, \mathbf{S}(y,z)) = \mathbf{S}(\mathbf{T}(x,y), \mathbf{T}(x,z))$$

for all $x, y, z \in [0,1]$ such that $\mathbf{S}(y,z) < 1$ We can label ([0, 1], S, T) a simulative semiring conditionally, The following characterization of continuous conditionally distributive semiring was obtained in (see Fig. 1)





PSEUDO-ANALYSIS AND GENERALIZATIONS

In mathematics, we are well used with the given comments:

1. Know the entire sequence term: 1, 5, 9, 13, 17, ...

2. Find a solution for the calculation of the inner angle of a normal n-side polygon.

3. Consider the chain of pseudo-analysis and generalizations

$$x_1 = \sqrt{2}$$

$$x_n = \sqrt{2} + x_{n-1}$$

Seeking an explicit nth term formula.

1. The sum of two consecutive integer numbers is strange.

2. The midpoint section between two sides of a triangle is parallel to the third side and is half as long.

3. The first by-product of an improved role is optimistic.

The generalization method is one of the strongest reasoning and its disintegration when we analyses the mathematical circumstance.

Pseudo-operations, pseudo-additive measures, pseudo-integrals

Let X be a set that is not vacant. Let A be a subset r-algebra of X. A fixed function (or interval semi closed) is a $\oplus$-decomposable measure if there hold $m(\emptyset) = 0$; and $m(A \cup B) = m(A) \oplus m(B)$ for $A, B \in \mathscr{A}$ such that $A \cap B = \emptyset$;. A $\oplus$-decomposable measure m is $\oplus$-measure if

$$m\left(\bigcup_{i=1}^{\infty} A_i\right) = \bigoplus_{i=1}^{\infty} m(A_i)$$

Hold every series $(A_i)_{i \in \mathbb{N}}$ *A* pair of disjoint sets:

The pseudo-integral structure based on $\oplus$-measure, denoted by

$$\int_X^{\oplus} f \odot \mathrm{d}m,$$

The construction of the Lebesgue integral is close. The pseudo-convolution and the pseudo-Laplace transformation can then be applied with various implementations. Let G be the Rn and * subset of a G binary switchable operation which is a cancellative semigroup with unit member e and

$$G_+ = \{x \mid x \in G, x \geqslant e\}$$

is a sub semigroup of G. We shall consider functions whose domain will be G.

Definition5. The very first two-function pseudo-convolution $f : G \to [a,b]$ and $h : G \to [a,b]$ with respect to a $\oplus$-measure m and $x \in G_+$ is given in the following way

$$f \star h(x) = \int_{G_+^x}^{\oplus} f(u) \odot \mathrm{d}m_h(v),$$

Where
$G_+^x = \{(u,v) | u * v = x, v \in G_+, u \in G_+\}$, $m_h = m$ in the case of sup-measure $m(A) = sup_{x \in A} h(x)$, in the case of

inf-measure $m(A) = \inf_{x\in A} h(x),$; and $\mathrm{d}m_h = h\odot \mathrm{d}m$ in the case of $\oplus$ -measure m, where $\oplus$ The Lebesgue measure has an additive generator g and g m. We also consider the second form of pseudo-convolution $(G,*)$ is a category, and the pseudo-integral is taken over whole group G:

$$f \star h(x) = \int_G^{\oplus} f(x*(-t)) \odot \mathrm{d}m_h(t),$$

Where (-t) is a special t and t reverse feature $x \in G.$

Note 1. If * is the normal addition to R and $G = \mathbb{R},$, And first second form pseudo-convolutions, $x \in \mathbb{R}^+$, are

$$(f \star h)(x) = \int_{[0,x]}^{\oplus} f(x-t) \odot \mathrm{d}m_h(t), \qquad (f \star h)(x) = \int_G^{\oplus} f(x-t) \odot \mathrm{d}m_h(t),$$

respectively.

Pseudo-delta feature is given

$$\delta_e^{\oplus,\odot}(x) = \begin{cases} \mathbf{1} & \text{for } x = e, \\ \mathbf{0} & \text{for } x \neq e, \end{cases}$$

Here zero is for 0 $\oplus$, 1 is the element unit for and e is the element zero for *.

Example 1. Let for * = + and $G = \mathbb{R}.$. For the semiring $([-\infty, \infty[, \max, +)$ from Section 2.3, the pseudo integral, with respect to sup-measure $m, m(A) = \sup_{x\in A} h(x),$, is given by

$$\int_{\mathbb{R}}^{\oplus} f \odot \mathrm{d}m = \sup_{\mathbb{R}}(f(x) + h(x))$$

and the first and second form pseudo-convolution of the functions f and h will be

$$(f \star h)(x) = \sup_{0\leq t\leq x}(f(x-t) + h(t)), \qquad (f \star h)(x) = \sup_{t\in\mathbb{R}}(f(x-t) + h(t)),$$

respectively. The pseudo-convolution unit member is the following pseudo-delta function

$$\delta_0^{\max,+}(x) = \begin{cases} \mathbf{1}(=0) & \text{if } x = 0, \\ \mathbf{0}(=-\infty) & \text{if } x \neq 0. \end{cases}$$

We only note here that the pseudo-convolution encompasses several essential foundational principles in numerous areas:

(1) The fundamental principle of probabilistic metric spaces, the function of triangles, is based on the first form pseudo-convolution.

(2) Fuzzy number arithmetic operations based on the extension theory of Zadeh, let T be an arbitrary but set t-norm, and * a binary operation on R. So, service * is applied to fluctuating numbers A and B

$$A *_{\mathbf{T}} B(z) = \sup_{x*y=z} \mathbf{T}(A(x), B(y))$$

for $z \in \mathbb{R}$.

(3) In the system theory pseudo-convolutions are as essential as classical convolution.

The pseudo-operations in the philosophy of pseudo-analysis have more generalizations. Firstly, since operations and are non-commutative and non-associative, and the (left) right distributivity over play a key function, see the first generalization of actual semiring structure. A representation theorem is obtained for such operations and a full characterization for generalized pseudo addition and pseudo-multiplication is given. An additional generalization includes symmetrizing the maximum and minimum, used to represent the utility functional integrally.

APPLICATIONS OF THE PSEUDO-ANALYSIS

Hamilton-Jacobi equation with non-smooth Hamiltonian

Here we consider the non-linear PDE, the Hamilton-Jacobi-Bellman equation.

$$\frac{\partial u(x,t)}{\partial t} + H\left(\frac{\partial u}{\partial x}, x, t\right) = 0,$$

In the control theory, Hamilton-Jacobi equations are especially relevant. The Hamilton-Jacobi equations in which the non-linear Hamiltonian H is not smooth are, sadly, typically the important versions, such as the absolute, min or max operation. Therefore, we cannot extend classical mathematical theory in those situations. There is the 'viscosity solution' method, that gives upper and lower solutions but not a classical solution, i.e. that its substitution into the formula reduces the identity equation. Using pseudo-analysis with generalized pseudo-convolution, solutions can be obtained which are interpreted in the formal sense described. We now expand the theory of pseudo-superposition to a more general situation.

Theorem 1 Let f be a R n function, with value of the form P seminal (min, +) or (min, maximum) and a practical function. $m_f : C_0^{cs}(\mathbb{R}^n) \to P$ is given by

$$m_f(h) = \int^{\oplus} f \odot dm_h = \inf_x (f(x) \odot h(x)).$$

Then

1) The mapping $f \mapsto m_f$ is a pseudo-isomorphism of the sub-module of higher semicontinuous features $(C_0^{c*}(\mathbb{R}^n))^*$

2) The space $(C_0^{c*}(\mathbb{R}^n))$ isometrically isomorphic with small function space, i.e. for all $m_{f_1}, m_{f_2} \in C_0^*(\mathbb{R}^n)$ we have

$$\sup_x d(f_1(x), f_2(x))$$

$$= \sup\{d(m_{f_1}(h), m_{f_2}(h)) : h \in C_0(\mathbb{R}^n), D(h, \mathbf{0}) \leq 1\}.$$

3) The mf1 and mf2 functions are equal, if and only if Clf1 = Clf2, where

$$Clf(x) = \sup\{\psi(x) : \psi \in C(\mathbb{R}^n), \psi \leq f\}.$$

We now evaluate the two Hamilton-Jacobi(- Bellman) Cauchy query

$$\frac{\partial u}{\partial t} + H\left(\frac{\partial u}{\partial x}\right) = 0, \quad u(x, 0) = u_0(x),$$

Where *x ∊ R* n allow R n and function *H: R n → R* allow R is convex (it is also constant with H limits). For control theory, the Hamiltonian H 's essential examples are highly nonlinear functions, such as max and. The pseudo-analysis technique prevents the use of the process named 'viscosity solution,' which does not have the exact solution. We are now utilizing pseudo-analysis approaches.

CONCLUSION

We also presented a segment named pseudo-analysis from the philosophy of generalized real analysis. A quick description of several significant applications in various fields is given. There are also several other significant applications where this modern method shades very different lights.

Therefore, the infinite dimension generalization of the theory of pseudo homogeneous non expansionary maps (which actually does not exist) which can be extended directly to the study of the price of derivative securities is necessary. This method, on the other side, which does not use either martingales or stochastic equations, makes the whole standard equipment theory suitable for the alternative pricing analysis.

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