A Study on Determinantal Representation of Stable Polynomials

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Abstract – In this paper you can find some findings which replace the "real-zero condition with properties known as the x replacement and Y replacement. We prove that in terms of identities and hermit matrices, we may always compose a deciding representation of a stable polynomial.

Keywords: Hermit Matrix, Stable Polynomial, Decisive Depiction

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INTRODUCTION

Multivariate polynomial stability is an important phenomenon that appears in a number of fields including analysis, electrical engineering and theory of control. The easiest way to decide if a specific bivariate is stable (no nulls inside the unit disc) is to look at the determinant (det(I + xX yY)) representation of the unit. The polynomial is stable if such a representation occurs, otherwise it is not. There is a defining representation and method to render this type representation for any bivariate, real-zero polynomial p(x,y), with p(0, 0) = 1,

$$p(x, y) = \det(I + xX + yY)$$

Where X and Y are Hermit matrices of the same dimension as P [1]. We are now interested in whether x-substitution and y-substitution properties will replace the condition "absolute zero." One of the problems is that, provided the polynomial p(x,y,z), p(x,y,1) is ideal for the replacement of the X and your alternative, and p(0, 0,1) = 1, Hermit matrices X and Y occurs such that there are no other matrices.

$$p(x, y, 1) = \det(I + xX + yY).$$

The following query may also be: p(x, y, 1) with p(0, 0, 1) = 1 has only actual zeros and p(x, 0, 1) and p(0, y, 1) have negative roots only where there is a certain positive sequence of X and Y only if p(x, y,1) = det(I + xX + yY). > The following question may be posed in the following question:

Note 1. The relation between the conjecture of Horn and the curve of Vinnikov is given by [3].

Note 2. There are many understandable (not easy) articles, such as [2], on the relation between Schur-Agler class and stable polynomials.

Definition 1.

If for all (x, y) $\in \mathbb{R}^2$ the only vector polynomial p(x,y)(t): = p (xt,yt) has only true zeros A bibliographic polynomial $p(x, y) \in \mathbb{R}[x, y]$ is considered an absolute nil polynomial

Definition 2. A bivariate polynomial $p(x, y) \in \mathbb{R}[x, y]$ satisfies *x*-substitution if for any α $\in \mathbb{R}$, *p*(α, *y*) has only real zeros.

p(x, y) follows the substitution property because p(x, y) fulfils the condition for x replacement and y replacement.A genuine zero-polynomial is a fact that does not inherently fulfil replacement properties. A true zero polynomial implies geometrically that the curve crosses with any line that goes through the root, and the replacement property means that the curve crosses both horizontal lines and vertical lines.

Lemma 0.1. Assume that f(x) is a degree-n polynomial with positive leaders and valid roots {a1, a2, . , an}.

Assume that g(x) is a degree n − 1 polynomial with a positive lead coefficient. If c1, If c1, c2,, cn ∈ R ,

That's it.

$$g(x) = c_1\frac{f(x)}{x - a_1} + c_2\frac{f(x)}{x - a_2} + \ldots + c_n\frac{f(x)}{x - a_n}$$

The origin of f is interlaced with g branches.

Furthermore, if f(x) roots are all true, then all f J(x) roots are correct and all f(x) roots are interlace, as f J(x) is =

$$\frac{1}{n}\frac{f(x)}{x-a_1}+\frac{1}{n}\frac{f(x)}{x-a_2}+\ldots+\frac{1}{n}\frac{f(x)}{x-a_n}$$

Definition 3. Denote P2 the bivalve polynomial range of grade n, which is consistent with x replacements and y replacements, and all homogeneous coefficients of the component of grade n.Determinantal Representation of Stable Polynomials

Theorem 0.2. If f (x), y) P2 and f (0, 0) = 1 are positive and all f (x, y) coefficients are positive, so those positive matrices X, Y are positive and f (x, Y) = det(I+ xX + y Y) are positive.

Lemma 0.3. If all f(x) = det(I + xX) coefficients are positive, so X is positive with its own values.

Corollary 0.4. If all f(x,y) = det(I + xX + yY) and f(0,y) are true, the substitution property is fulfilled by both f(x,y) and f (0), so both of X and Y are positive. Y is positive, then both f(x,0) and f (0,y).

Lemma 0.5. If f (x, y) ∊ P2 and all f (x, y) coefficients are positive, then f (x, αx) satisfies the replacement characteristic of all α R. (Hencef (x, y) is a true zero polynomial and Hermetic matrices X and Y have been formed to f (x, y) = det(I + xX + yY).

We also realise that (1)f(x,y) is a valid polynomial zero and all coefficients are positive only if and only when f(x,y) = det(I+xX +yY) where X,Y is positive and (2) P2, if f(x,y), = dét(I +xX + yY) | ∊ P2 ,

Lemma 0.6. *P(x, y) is a det (I + xX + yY) representation where X, Y are Hermit, if p(x, y) is true polynomial zero.*

***Proof**.* "⇐" Please be noted that p(xt,yt) = det(I+ (xX+yY)t) where the Hermit is xX+yY for every x,y R. Record all xX + yY by λ1, λ2, . , λn. own values . So the p(xt, yt) zeros is 1 for all I = 1, 2,. n Ç 0. n λi Ç 0. To be sure it'll diagonalizexX + yY = UΛU∗ lumber to lumber to lumber where U is unitary, Λ = diag(λ1, . . . , λn).∗. Then you have to.

Then $p(xt,yt)=\det(UIU^*+tU\Lambda U^*)=\det(U)\det(I+t\Lambda)\det(U^*)$.

Hence

$$p(xt,yt)=0\Leftrightarrow\det(I+t\Lambda)=(1+\lambda_1 t)\cdots(1+\lambda_n t)=0.$$

"⇐" It is due to [1]

The Vinnikov curve, where both X and Y are positive, is noted 3.p (x, y) = det(I + xX +yY).

Lemma 0.7When a real zero p(x, y) = det(I+xX+yY) fulfils one of the following requirements, it has x-replacement and y-replacement properties respectively.

1. The positive matrices are both X and Y definite.

2. Move X and Y.

Proof. Since X is definite positive, it has a certain positive square root. 'With (1).' Let B2 = X be definite where B is positive. Det(X) performance factoring

$$p(x,y)=\det(X)\det(xI+yB^{-1}YB^{-1}+X^{-1})$$

Has any y = α ∊ R. actual roots. First, det(Y) factoring generates all actual roots of the p(α, y).When X and Y traffic, they may be diagonalized at the same moment. 'With (2)' Via this process p(i, y) = det(U) det(I + xΛ +yΦ) det(U∗

$$p(x,y)=0\Leftrightarrow(1+x\lambda_1+y\phi_1)\cdots(1+x\lambda_n+y\phi_n)=0.$$

Lemma 0.8. If X and Y are described positively in X n matrices, the p(x, y) = det(I + xX + yY) substitution satisfies both x and y.

Proof. As X is certainly optimistic, it has a definitely positive square root. Let B2 = X be definite where B is positive. Det(X) performance factoring

$$p(x,y)=\det(X)\det(xI+yB^{-1}YB^{-1}+X^{-1})$$

Has any y = α ∊ R actual roots. First, det(Y) factoring generates all actual roots of the p(α, y).

Theorem 0.9. Let p(x,y) be a true zero polynomial with a defining dep (I+xx+yY), in which X,Y are Hermitian. Then p(x, y) satisfies x and y substitution only if there is one of these parameters

1. X and Y are certainly positive

2. Shuttle X and Y

Proof. The data is conveniently collected by integrating lemmas 0.7 and 0.8.

We are also providing explanations that will help audiences to comprehend.

Examples

Example 1. Let $p(x, y) = 1 + 3x + 5y + 2x^2 + 7xy + 5y^2$. Then $p\tilde{\ }_y(t) = t^2 + (3 + 5y)t + (5y^2 + 7y + 2)$. We get

$$H(y)=\begin{bmatrix}2 & -3-5y\\ -3-5y & 15y^2+16y+5\end{bmatrix}.$$

Next, factor $H(y) = Q^*(y)Q(y)$. We write this factorization as follows.

$$H(y) = \begin{bmatrix} \sqrt{2} & 0 \\ -\frac{3\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}y & \sqrt{5/2y^2 + y + 1/2} \end{bmatrix} \begin{bmatrix} \sqrt{2} & -\frac{3\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}y \\ 0 & \sqrt{5/2y^2 + y + 1/2} \end{bmatrix}.$$

Notice that since √2 and $5/2y^2 + y + 1/2 = 5/2[(y + 1/5)^2 + (2/5)^2] > 0$ for all $y \in$ R. Thus $Q(y)$ is positive definite for all $y \in$ R. The polynomial p was constructed using X

$$= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}.$$

Example 2. Let $p(x, y) = 2x^2 + 7xy + 3x + 5y^2 + 5y + 1$, then $p\check{}_y(t) = t^2 + 5ty + 3t + 5y^2 + 7y + 2 = t^2 + (5y + 3)t + (5y^2 + 7y + 2)$.

So

$$H(y) = \begin{pmatrix} 2 & -5y - 3 \\ -5y - 3 & 15y^2 + 16y + 5 \end{pmatrix}$$

$$Q(y) = \begin{pmatrix} \sqrt{2} & -\frac{5\sqrt{2}}{2}y - \frac{3\sqrt{2}}{2} \\ 0 & \sqrt{\frac{5}{2}}(y + \frac{1+2i}{5}) \end{pmatrix}$$

where (2, 2)-entry is obtained by solving

$$\frac{5}{2}(y^2 + \frac{2}{5}y + \frac{1}{5}) = 0$$

$$Q^*(y) = \begin{pmatrix} \sqrt{2} & 0 \\ -\frac{5\sqrt{2}}{2}y - \frac{3\sqrt{2}}{2} & \sqrt{\frac{5}{2}}(y + \frac{1-2i}{5}) \end{pmatrix}.$$

$$Q^{-1}(y) = (\sqrt{5}(y + \frac{1+2i}{5}))^{-1} \begin{pmatrix} \sqrt{\frac{5}{2}}(y + \frac{1+2i}{5}) & \frac{5\sqrt{2}}{2}y + \frac{3\sqrt{2}}{2} \\ 0 & \sqrt{2} \end{pmatrix}.$$

$$M(y) = Q(y)C(y)Q(y)^{-1} = \begin{pmatrix} -\frac{5y}{2} - \frac{3}{2} & \frac{5y+1-2i}{2\sqrt{5}} \\ \frac{5y+1+2i}{2\sqrt{5}} & -\frac{5y}{2} - \frac{3}{2} \end{pmatrix}$$

Thus, the stable polynomial is

$$p(x, y) = \det(I_2 + x\begin{pmatrix} \frac{3}{2} & \frac{1-2i}{2\sqrt{5}} \\ \frac{1-2i}{2\sqrt{5}} & \frac{3}{2} \end{pmatrix} + y\begin{pmatrix} \frac{5}{2} & \frac{-\sqrt{5}}{2} \\ \frac{-\sqrt{5}}{2} & \frac{5}{2} \end{pmatrix}).$$

CONCLUSION

It is a very complicated topic to describe stable polynomials with actual zero or complex nulls. We offer some findings that "real-zero" can be replaced by so-called x-substitution and y-substitution properties. These two features help us to write the picture quickly. With steady polynomials with additional variables, we can research this further.

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