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Static Spherically Symmetric Charged Fluid Sphere in Einstein-Cartan Maxwell Theory

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Abstract – The present paper deals with charged fluid sphere in Einstein-Cartan theory. In this paper, we have studied the interior field of a static spherically symmetric charged fluid distribution with spin. Assuming that the spin of the individual particles compositing the fluids are all aligned along radial direction, we have obtained solutions by choosing metric potential α(r) and β(r) on different suitable forms or conditions. Pressure and density have been also calculated for the distribution and the physical constants appearing in the solution have been evaluated by matching the solutions to the Reissner-Nordstrom metric at the boundary. It is found that for a realistic model p > 0, p > 0, which will impose further restrictions on our solutions.

Key Words – Charge, Fluid Sphere, Metric Potential, Boundary Conditions, Pressure Matter Density.

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1. INTRODUCTION

Various authors have made their attempts to investigate the problem of charged fluid spheres in E-C theory. Nduka [7], Singh and Yadav [10], Prasanna [8], Kopenzynski [3, 4] and Raychaudhari [9] have considered the generalization of Maxwell's equations in space having torsion but this idea leads to a breakdown in the gauge invariance and charge conservation principle. However, Raychaudhari [9] and Nduka [7] have taken the equation in a form so as to mepressure the charge conservation principle. With this formulation Raychaudhari [9] has investigated the possibility of bounce in the pressure of magnetic field for Bianchi type I universe with p = 0 and p = 0. Further Singh and Yadav [10] have discussed the static charged fluid sphere in E-C theory and have found that the pressure is discontinuous at the bounding of the fluid sphere. Some other workers in this line are Krori et al. [3], Mehra and Gokhroo [6]. Suh [12], Som and Bedron [11]. Yadav and Prasad [13]. Thomas, Maurya, Pant, Patel, Ratanpal, et al. [14-21].

In the paper, we have studied the interior field of a static spherically symmetric charged fluid distribution with spin. Assuming that the spins of the individual particles compositing the fluid area all aligned along radial direction, we have obtained solutions by choosing metric potential α(r) and β(r) in different suitable forms. Pressure and density have been also calculated for the distribution and physical constants appearing in the solution have been evaluated by matching the solutions to the Reissner-Nordstrom metric at the boundary.

2. THE FIELD EQUATION

The Einstein Cartan Maxwell equations are

(2.1) $$R^i_j - \frac{1}{2}R\delta^i_j = -8\pi T^i_j$$

(2.2) $$Q^k_{ij} - \delta^k_i Q^l_{jl} - \delta^k_j Q^l_{il} = -8\pi S^k_{ij}$$

(2.3) $$\left[-g^{1/2}F^{ij}\right]i = (-g)^{1/2}J^i = (-g)^{1/2}\sigma u^i$$

(2.4) $$\left[F_{ij};k\right] = 0$$

Where R_{ij} is the Ricci Tensor of asymmetric connection and also the energy momentum tensor t_{ij} is not symmetric, F_{ij} is the electromagnetic field tensor, Q^{ij} is torsion tensor, S^{ij} is spin tensor, □ is charge density and j[] is current four vector (we have set C and gravitational constant also equal to unity)

Now we have se the static spherically symmetric metric

(2.5) $$ds^2 = e^\beta dt^2 - e^\alpha dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

Where α and β are functions of r only.

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For the system under study the symmetric energy momentum tensor T^i_j and E^i_j for matter and electromagnetic field respectively as

(2.6) $$T^{-i}_j = T^i_j + E^i_j$$

where,

$$T^i_j = (\rho + p)u^i u_j - \delta^i_j p$$

$$E^i_j = -F_{jY}F^{iY} + \frac{1}{4}\delta^i_j F_{lm}F^{lm}$$

we use comoving co-ordinates so that

$$u^1 = u^2 = u^3 = 0 \quad u^4 = e^{-\beta/2}$$

The non-vanishing components of the energy momentum tensor are

$$T^1_1 = T^2_2 = T^3_3 = -p \text{ and } t^4_4 = \rho$$

We can then write the field equations

(2.7) $$8\pi\bar{p} - E = e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

(2.8) $$8\pi\bar{p} + E = e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} + \frac{\beta' - \alpha'}{2r}\right)$$

(2.9) $$8\pi\bar{\rho} + E = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

Here following Hehl [1, 2], we have defined effective density $\bar{\rho}$ and effective pressure $\bar{p}$ as

(2.10) $$\bar{\rho} = \rho - 2\pi k^2 \text{ and } \bar{p} = p - 2\pi k^2$$

(2.11) $$k = He^{-\beta/2}$$

Here H is constant and dashes denotes differentiation with respect to r.

(2.12) $$E = -F_{41}F^{41}$$

and

(2.13) $$4\pi\bar{\sigma} = \left[\frac{dF^{41}}{dr} + \frac{2F^{41}}{r} + \left(\frac{\alpha' + \beta'}{2}\right)F^{41}\right]e^{\beta/2}$$

3. SOLUTION OF THE FIELD EQUATIONS

Using the equation (2.7) – (2.9), we get

(3.1) $$8\pi\bar{p} = \frac{e^{-\alpha}}{2}\left(\frac{3\beta'}{2r} + \frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} - \frac{\alpha'}{2r} + \frac{1}{r^2}\right) - \frac{1}{2r^2}$$

(3.2) $$E = \frac{e^{-\alpha}}{2}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} - \frac{\alpha'}{2r} - \frac{\beta'}{2r} + \frac{1}{r^2}\right) + \frac{1}{2r^2}$$

(3.3) $$8\pi\bar{\rho} = e^{-\alpha}\left(\frac{3\alpha'}{4r} - \frac{\beta''}{4} + \frac{\alpha'\beta'}{8} - \frac{\beta'^2}{8} + \frac{\beta'}{4r} - \frac{1}{2r^2}\right) + \frac{1}{2r^2}$$

Equation (3.1) and (3.3) using (2.10) gives pressure and density as

(3.4) $$8\pi p = \frac{e^{-\alpha}}{2}\left(\frac{3\beta'}{2r} + \frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} - \frac{\alpha'}{2r} + \frac{1}{r^2}\right) - \frac{1}{2r^2} + 16\pi^2 k^2$$

(3.5) $$8\pi\rho = e^{-\alpha}\left(\frac{5\alpha'}{4r} - \frac{\beta''}{4} + \frac{\alpha'\beta'}{8} - \frac{\beta'^2}{8} + \frac{\beta'}{4r} - \frac{1}{2r^2}\right) + \frac{1}{2r^2} + 16\pi^2 k^2$$

With three equation (2.7)- (2.9) in five variable (p, E, p, α, β) the system determinate, we require two more equations or relations. For this we choose α and β as

(3.6) $$\alpha = ar^2 + c$$

(3.7) $$\beta = dr^2 + k_1$$

(3.8) $$k^2 = H^2 \exp\{-(dr^2 + k_1)\}$$

(3.9) $$16\pi p = 32\pi^2 H^2 \exp\{-(dr^2 + k_1)\} + \frac{1}{2}e^{-(ar^2+c)}\left[4d - a - dr^2(a - d) + \frac{1}{r^2}\right] - \frac{1}{2r^2}$$

(3.10) $$16\pi p = 32\pi^2 H^2 \exp\{-(dr^2 + k_1)\} + \frac{1}{2}e^{-(ar^2+c)}\left[3a + rd^2(a - d) - \frac{1}{r^2}\right] + \frac{1}{2r^2}$$

(3.11) $$E = -\frac{1}{2}e^{-(ar^2+c)}\left[d(1 - rd^2) + a(1 + rd^2) + \frac{1}{r^2}\right] + \frac{1}{2r^2}$$

(3.12) $$4\pi\sigma = \left[\frac{dF^{41}}{dr} + \frac{2F^{41}}{r} + r(a + d)F^{41}\right]e^{\frac{(dr^2+k_1)}{2}}$$

Also using boundary condition at r = r_0, we have

$$e^{-(ar_0^2+c)} = \left(1 - \frac{2M}{r_0} + \frac{Q_0^2}{r_0^2}\right) \quad (3.13)$$

$$e^{-(dr_0^2+k_1)} = \left(1 - \frac{2M}{r_0} + \frac{Q_0^2}{r_0^2}\right) \quad (3.14)$$

$$2dr_0 e^{-(dr_0^2+k_1)} = 2\left(\frac{M}{r_0^2} - \frac{Q_0^2}{r_0^3}\right) \quad (3.15)$$

$$H^2 = \frac{e^{(dr_0^2+k_1)}}{32\pi^2}\left\{\left(16\pi\rho_0 - \frac{1}{2r_0} - \frac{1}{2}e^{-(ar_0^2+c)}\right)\right. \quad (3.16)$$

$$\left.\left[3a + r_0 d^2(a-d) - \frac{1}{r_0^2}\right]\right\}$$

4. DISCUSSION

In this paper we have studied the interior field of a static spherically symmetric charged fluid distribution with spin. Assuming that the spins of the individual particles compositing the fluid area all aligned along radial direction, we have obtained solutions by choosing metric potentials α(r) and β(r) in different suitable forms. Pressure and density have been also calculated for the distribution and physical constants appearing in the solution have been evaluated by matching the solutions to the Reissner-Nordstrom metric at the boundary. Further for a realistic model p > 0, p > 0 which will impose further restrictions on our shlutions.

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